Section 6.1: Antiderivatives
Section Objectives:

- Know the definition of an antiderivative and what it means to be a general antiderivative.
- Know the symbol for indefinite integrals and that this is the same as the general antiderivative.
- Know the antiderivative rules listed below.
  1. $\int kf(x)\,dx = k\int f(x)\,dx$
  2. $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$
  3. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad n \neq -1$
  4. $\int e^x\,dx = e^x + C$
  5. $\int \frac{1}{x}\,dx = \ln|x| + C$
- Use antiderivatives to find cost and revenue functions from marginal cost and revenue (and other similar examples).

Practice Problems

1. Evaluate the following antiderivatives:

   (a) $\int 3x^2 + 2x\,dx$

   $$3\cdot\frac{x^3}{3} + 2\cdot\frac{x^2}{2} + C = \boxed{x^3 + x^2 + C}$$

   Check deriv: $3x^2 + 2x$ ✓

   (b) $\int \sqrt{x} + \frac{1}{x^2}\,dx$

   $$= \int x^{1/2} + x^{-2}\,dx$$

   $$= \frac{x^{3/2}}{3/2} + \frac{x^{-1}}{-1} + C = \boxed{\frac{2}{3}x^{3/2} - \frac{1}{x} + C}$$

   Check derivative:

   $x^{1/2} + \frac{1}{x^2}$ ✓

   (c) $\int \frac{1}{x} + 3e^x\,dx$

   $$= \boxed{\ln|x| + 3e^x + C}$$

   Check derivative

   $\frac{1}{x} + 3e^x$ ✓

(d) $\int \frac{t^4+4}{t^2}\,dt = \int t^2 + \frac{4}{t^2}\,dt = \int t^2 + 4t^{-2}\,dt$

$= \frac{t^3}{3} + \frac{4t^{-1}}{-1} + C$

$= \boxed{\frac{1}{3}t^3 - 4/t + C}$

check derivative

$t^2 + \frac{4}{t^2}$ ✓

(e) $\int (x+1/x)(x+3)\,dx$

$= \int x^2 + 3x + 1 + \frac{3}{x}\,dx$ $\quad \frac{3}{x} = 3x^{-1}$

$= \boxed{\frac{x^3}{3} + \frac{3}{2}x^2 + x + 3\ln|x| + C}$

Check derivative

$x^2 + 3x + 1 + \frac{3}{x}$ ✓

2. Is $F(x) = \ln(2x^2+1) + C$ an antiderivative of $f(x) = \frac{1}{2x^2+1}$? Why or why not?

This is the same as asking "is the derivative of $\ln(2x^2+1)+C$ equal to $\frac{1}{2x^2+1}$

$\frac{d}{dx}\left(\ln(2x^2+1)+C\right) = \frac{1}{2x^2+1} \cdot 4x$ (chain rule) so No, it's Not.

3. Kyle finds that for his designer coffee bean business that the marginal revenue is $30-2x$ where x is in bags of beans produced and sold. Find his revenue function. How much should he produce to maximize revenue?

$R'(x) = 30 - 2x$

so $R(x) = \int (30-2x)\,dx = 30x - x^2 + C$

Since $R(0) = 0$, $C = 0$, so $R(x) = 30x - x^2$

↑since \$0 revenue for selling 0 items.

To max revenue

$R'(x) = 0$

$30 - 2x = 0$

$x = 15$

R' + + + | − − −

So $x = 15$ is max.

4. Mia is selling handmade watches. The marginal cost function for a watch is given by $150 - 1/2e^x$. The fixed costs are \$300. What is the cost function?

$C'(x) = 150 - \frac{1}{2}e^x$

So $C(x) = \int 150 - \frac{1}{2}e^x\,dx$

$C(x) = 150x - \frac{1}{2}e^x + C$

$C(0) = 0 - \frac{1}{2} + C = 300$

$C = 299.5$

So $\boxed{C(x) = 150x - \frac{1}{2}e^x + 299.5}$

More Practice from Textbook 6.1: You should do as many problems from each set (1-38, 39-55, 56-57), as needed until you are comfortable with these techniques. 39-55 are good practice for application problems.