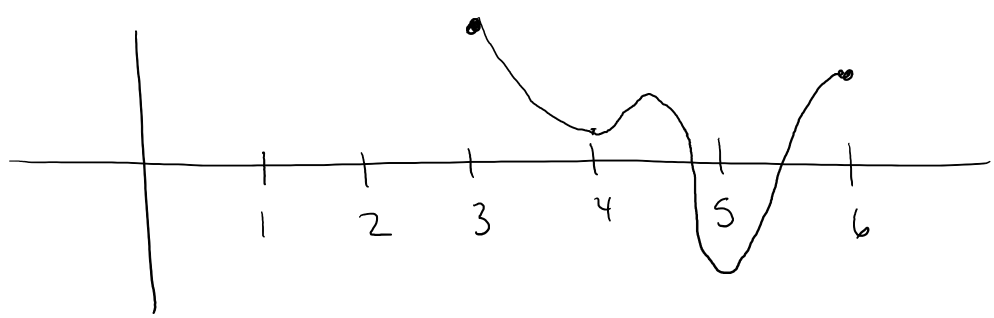
Section 5.5: Absolute Extrema
Section Objectives:

- Know the definition of an absolute minimum and maximum
- Know what conditions guarantee a function has an absolute minimum and maximum on an interval.
- Know how to find the absolute extrema of a function on a closed interval
- Use maximization techniques to solve word problems

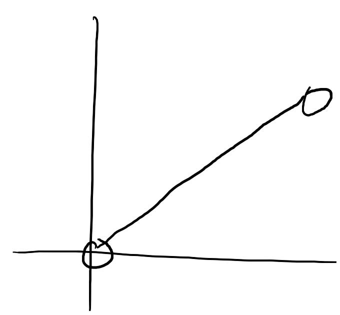
Practice Problems

1. Sketch the graph of a function on the domain $[3, 6]$ with a absolute maximum at $x = 3$ an absolute minimum at $x = 5$ and a local minimum at $x = 4$.



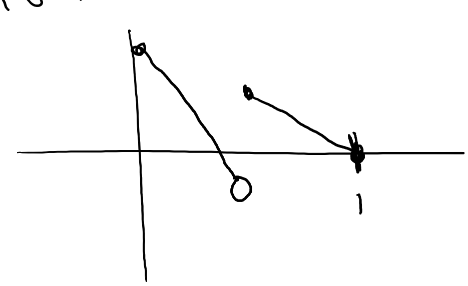
2. Give an example of a continuous function and an interval where that function has neither an absolute maximum or minimum.

let $f(x) = x$ on $(0,1)$
no min or max since $x=0$ and $x=1$ not included in interval



3. Give an example of a function on $[0, 1]$ that has a absolute maximum but no absolute minimum.

Since it's on a closed interval but has no minimum it must not be a continuous function



4. Find the absolute maximum and minimum values of $f(x) = x^3 - 3x^2 - 2$ on $[-2, 2]$

$$f'(x) = 3x^2 - 6x$$
$$= 3x(x-2)$$
$$x = 0,\ x = 2$$

x	$f(x)$
-2	$-8-12-2 = -22$
0	-2
2	$8-12-2 = -6$

absolute max of -2 at $x=0$

absolute min of -22 at $x=-2$

5. Find the absolute maximum and minimum values of $f(x) = \sqrt{x^2+1}$ on $(-\infty, \infty)$

$$f'(x) = \frac{1}{\cancel{2}}(x^2+1)^{-1/2}(\cancel{2}x) = \frac{x}{\sqrt{x^2+1}}$$

$f'(x) = 0$ when $x=0$, never undefined

* f' : $- - - - - - -$ | $+ + + + + +$

f : $\searrow \searrow \searrow$ 0 $\nearrow \nearrow \nearrow \nearrow$

* since not on closed interval, must consider entire function

$\lim_{x \to \pm\infty} \sqrt{x^2+1} = \infty$, so no absolute max

absolute min at $\boxed{x=0}$

value of $\boxed{1}$

6. It has been estimated that a rumor spreads at a rate R that is proportional both to the ratio r of individuals who have heard it and to the ratio $(1-r)$ of those who have not. Thus $R = kr(1-r)$, where k is a positive constant. For what value of r does the rumor spread the fastest?

Want to maximize R (product rule)

$$R' = kr(-1) + k(1-r)$$
$$= -kr + k - rk$$
$$= k(1-2r) = 0 \text{ when } r = 1/2$$

R' + + + + | − − − −

R ↗ ↗ 1/2 ↘ ↘ ↘

max at $x = 1/2$, $R = k/4$

7. Goal: Find two numbers whose sum is 24 and whose product is maximum.

(a) Call the two numbers x and y. What is the quantity we want to maximize?

maximize product : xy

(b) We only know how to maximize functions of one variable. Use the fact that the sum of the two number is 24 to write y in terms of x.

$$x + y = 24 \Rightarrow \boxed{y = 24 - x}$$

(c) Use the previous part to rewrite the function we want to maximize so that it only has one variable.

$$p = xy = x(24-x)$$
$$= 24x - x^2$$

(d) Use the techniques we learned in the section to find the maximum value of this function and the values of x and y.

$$p' = 24 - 2x = 0$$
$$\Rightarrow x = 12$$

p' + + + + | − − − −

↗ ↗ ↗ 12 ↘ ↘ ↘

max at $x = 12$, $y = 12$
value : $12(12) = 144$

More Practice from Textbook 5.5 You should do as many problems from each set (1-6, 7-10, 11-28, 29-30, 31-38, 43-52), as needed until you are comfortable with these techniques. 43-52 are good practice for application problems.