

Section 5.3: Limits at Infinity

Section Objectives:

- Know the definition of a horizontal asymptotes.
- Know how to algebraically find the limit at infinity of a rational function (by dividing by the highest power of x^* on numerator and denominator).
- Know how to algebraically find the limit at infinity of a polynomial by considering the dominant (highest power) term.

*Note: when deciding what power to divide by, some people divide by the highest overall power of x (on either the numerator or denominator) while other people divide by the highest power present in the denominator. Both of these methods are valid and will produce the proper result.

Practice Problems

1. In each of the following questions, plug in large values of x (or large negative values, where appropriate) to guess the limit. Then use algebraic techniques to evaluate the limit.

(a) $\lim_{x \rightarrow \infty} \frac{3x^2 + 4x}{5x^2 + 3x}$

(b) $\lim_{x \rightarrow -\infty} \frac{x - 2}{x^2 + 4}$

(c) $\lim_{x \rightarrow \infty} \frac{x^2 + 3x}{\sqrt{x} + 1}$

2. Let $f(x) = -3x^n + 2x^{n-1} + 3x + 1$ where n is a whole number greater than 2. What is $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$. Explain your reasoning. Your answer will depend on n .

3. Let $f(x) = \frac{2x^2 + 3x - 5}{x^2 + x - 6}$.

(a) Are there horizontal asymptotes? Clearly explain your reasoning.

(b) Recall that we have a vertical asymptote at $x = a$ if $\lim_{x \rightarrow a} f(x) = \pm\infty$. Are there any vertical asymptotes of this function?

(c) Use the first derivative to determine when this function is increasing and when it is decreasing.

(d) The second derivative of $f(x) = \frac{2(x^3 + 21x^2 + 39x + 55)}{(x^2 + x - 6)^3}$ It is equal to 0 at $x \approx -19.1$. Where is the function concave up and concave down? (Hint: a function can change concavity where the second derivative is 0 OR undefined.)

(e) Use the information above to sketch a graph of the function.

More Practice from Textbook 5.3: You should do as many problems from each set (1-18, 19-32, 33-37), as needed until you are comfortable with these techniques. 19-32 are good practice for application problems.