Section 5.2: The Second Derivative
Section Objectives:

- Know how to find the second derivative.
- Know what it means for a functions to be concave up or concave down (both in terms of the first derivative and the graph).
- Know how to use the second derivative to tell if a function is concave up or concave down.
- Know the definition of an inflection point of a function.
- Know how to use the second derivative to determine if a function has a minimum or maximum at a place where the derivative is 0.
- Know how to tell when an economy of scale exists.

Practice Problems

1. Sketch the graph of a function on the domain $[-5, 5]$ which satisfies all the conditions below.
   - increasing on $[-5, -3]$ and $[3, 5]$
   - decreasing on $[-3, 3]$
   - concave down on $[-5, 0]$
   - concave up on $[0, 5]$.

2. If $f'(4) = 0$ and $f''(4) = 5$, what can we say about f at $x = 4$? Explain your reasoning.

3. Let $f(x) = 3x^4 - 4x^3 + 1$. Find the intervals where $f(x)$ is increasing, decreasing, concave up and concave down. Find all relative extrema and inflection points. Use these to sketch a graph of the function.

4. Let $f(x) = e^{-x^2}$. Find the intervals where $f(x)$ is increasing, decreasing, concave up and concave down. Find all relative extrema and inflection points. Use these to sketch a graph of the function.

5. We have an economy of scale if the marginal cost ($C'(x)$) is decreasing as the number of units produced increases. What does this tell us about $C''(x)$? About $C(x)$?

6. If the cost function of a firm is given by $C(x) = -0.1x^2+2x+5$, is this firm experiencing an economy of scale? Explain your reasoning.

More Practice from Textbook 5.2: You should do as many problems from each set (1-6, 7-12, 13-20, 21-24, 25-36, 37-41, 45-62), as needed until you are comfortable with these techniques. 45-62 are good practice for application problems.