Section 5.1: The First Derivative
Section Objectives:

- Know how to use the first derivative to tell if a function is increasing or decreasing.
- Know the definition of a critical value of a function and how they can be used to find where a function is increasing or decreasing.
- Know the definition of relative (or local) extrema, relative(or local) minimum and relative(or local) maximum and how to find them.

Practice Problems

1. Let $f(x) = 1/3x^3 - 12x + 3$. Find the intervals where f is increasing and decreasing, the relative extrema and sketch a graph of f.

$f'(x) = x^2 - 12$

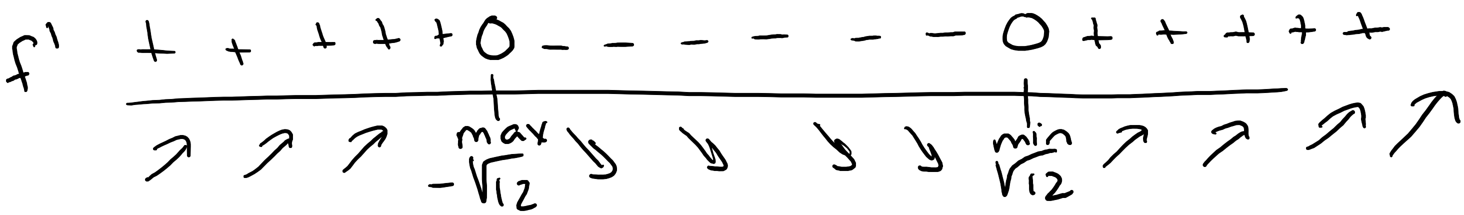
$f'(x) = 0$ when $x^2 - 12 = 0$

$\Rightarrow x^2 = 12, \; x = \pm\sqrt{12}$

when $x < -\sqrt{12}$ $\quad x^2 - 12 > 0$ (try $x=-10$) so f inc ↗

$-\sqrt{12} < x < \sqrt{12}$ $\quad x^2 - 12 < 0$ (try $x=0$) so f dec ↘

when $x > \sqrt{12}$ $\quad x^2 - 12 > 0$ (try $x=10$) so f inc ↗



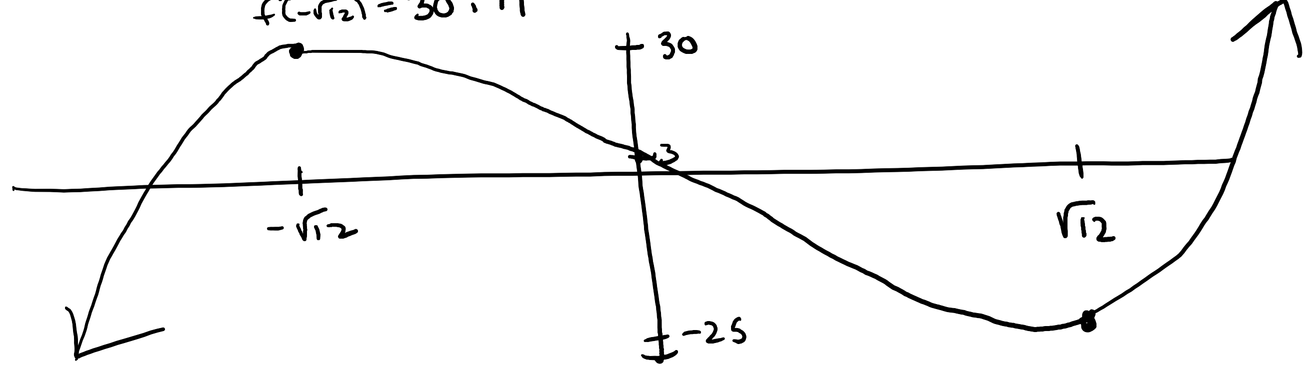
f increasing $(-\infty, -\sqrt{12}) \cup (\sqrt{12}, \infty)$

f decreasing $(-\sqrt{12}, \sqrt{12})$

relative max at $x = -\sqrt{12}$ $\quad$ relative min at $x = \sqrt{12}$

$f(-\sqrt{12}) = 30.71$ $\quad$ $f(\sqrt{12}) = -24.71$

$f(0) = 3$



2. Let $f(x) = \dfrac{x^2}{e^x}$. Find the intervals where f is increasing and decreasing, the relative extrema and sketch a graph of f.

$$f(x) = \frac{x^2}{e^x}, \quad f'(x) = \frac{e^x(2x) - x^2(e^x)}{(e^x)^2} = \frac{2x - x^2}{e^x} = \frac{x(2-x)}{e^x}$$

where is $f'(x) = 0$ or undefined?

$e^x \neq 0$ so never undefind.

$$f'(x) = \frac{x(2-x)}{e^x} = 0 \Rightarrow x(2-x) = 0$$

$\Rightarrow x = 0, x = 2$

try $x=-1$ try $x=1$ try $x=3$

f': $- - - - 0 + + + + + + + 0 - - - - - -$

f: ↘ ↘ ↘ 0 min ↗ ↗ ↗ ↗ 2 max ↘ ↘ ↘ ↘

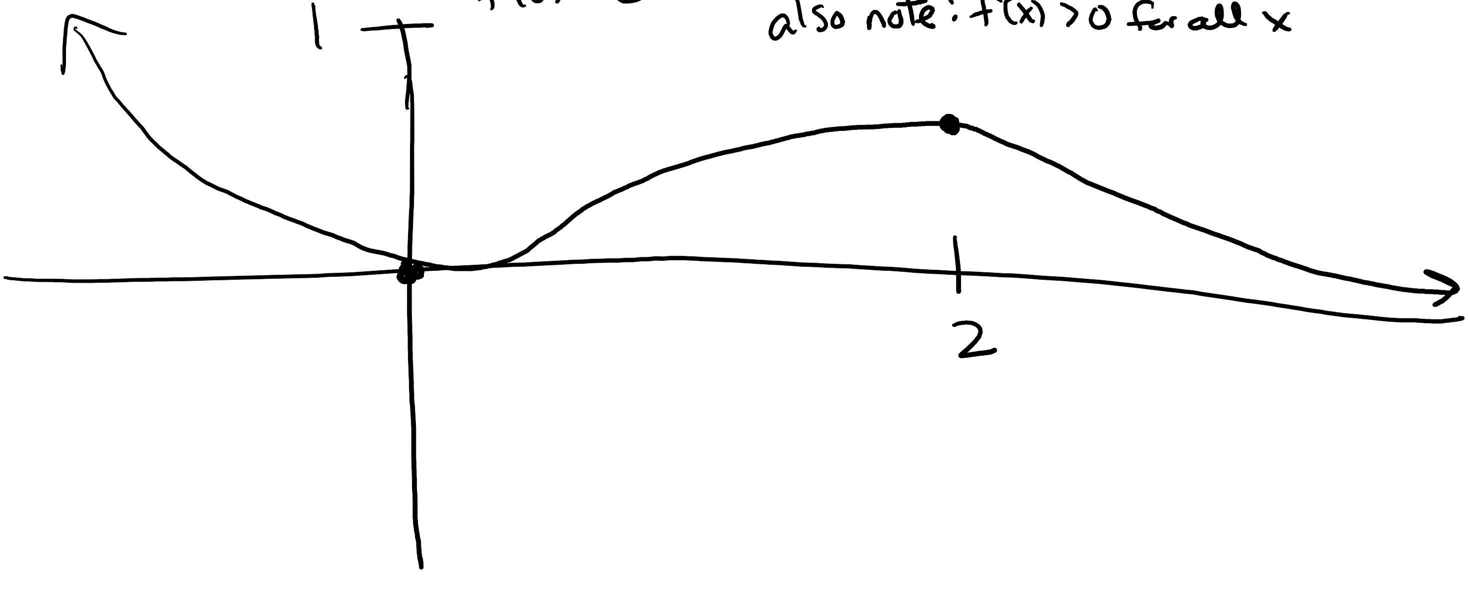
f decreasing $(-\infty, 0) \cup (2, \infty)$

f increasing $(0, 2)$

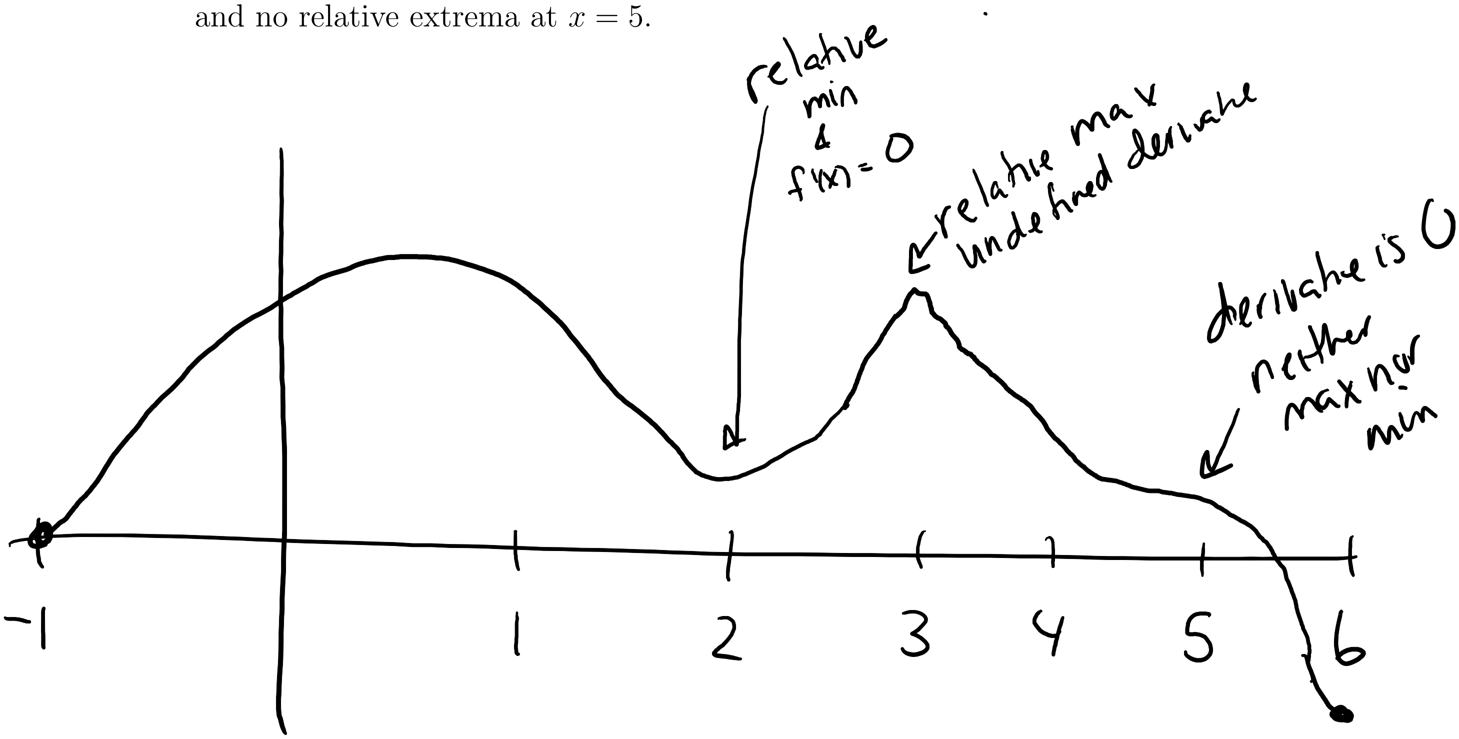
relative min at $x = 0$, $f(0) = 0$

relative max at $x = 2$, $f(2) = \frac{4}{e^2} \approx .54$

also note: $f(x) > 0$ for all x



3. Draw a function of the domain $[-1, 6]$ whose derivative is 0 at $x = 2$ and 5 and undefined at $x = 3$ and had a relative minimum at $x = 2$ a relative maximum at $x = 3$ and no relative extrema at $x = 5$.



4. The profit function for a lumber company is given by $P(x) = 3e^{-x^2+4x-4}$ where P is in thousands of dollars and x is tons of lumber sold. How much lumber should they sell to maximize profit?

$$P'(x) = 3e^{-x^2+4x-4}(-2x+4)$$

never 0, always positive so

$P'(x) = 0$ when $-2x + 4 = 0 \Rightarrow x = 2$.

$P'(x)$ try $x=0$ try $x = 3$

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P ↗ ↗ ↗ 2 ↘ ↘ ↘ ↘ ↘

max

maximum profit @ $X = 2$ tons of lumber sold.

More Practice from Textbook 5.1: You should do as many problems from each set (1-6, 7-12, 13-42, 49-71, 72-90, 91-96 37-44, 45-48), as needed until you are comfortable with these techniques. 49-71 are good practice for application problems.