

## Section 4.5: Elasticity of Demand

### Section Objectives:

- Know the formula for elasticity of demand.
- Know the interpretation of elasticity of demand and how to use it to answer questions.
- Know what it means for price to be elastic, unit elastic and inelastic and what this means about the price.
- Know how to tell if revenue is increasing, decreasing or maximized using elasticity.

### Practice Problems

1. Jake is selling boxes of pens. His demand function is  $p(x) = -1/2x + 5$ .

- (a) We want to rewrite the demand function so that it gives us  $x$  as a function of  $p$ . Do this by solving the demand function for  $x$ .

$$p = -1/2x + 5$$
$$p - 5 = -1/2x$$

$$x = -2p + 10$$

- (b) Find the elasticity of demand.

$$\epsilon = - \frac{dx}{dp} \cdot \frac{p}{x} \quad \frac{dx}{dp} = -2$$

$$\epsilon = \frac{-(-2)(p)}{-2p + 10} = \frac{2p}{10 - 2p}$$

- (c) Evaluate the elasticity of demand when  $p = 3$ . Interpret your answer, include is price is elastic, inelastic or unit elastic.

$$\epsilon(3) = \frac{6}{10-6} = \frac{6}{4} = \frac{3}{2} = 1.5$$

Since  $\epsilon > 1$  price is elastic.

a 2% increase in price leads to about a 3% decrease in demand.

- (d) Evaluate the elasticity of demand when  $p = 1$ . Interpret your answer, include if price is elastic, inelastic or unit elastic.

$$\epsilon(1) = \frac{2}{10-2} = \frac{2}{8} = \frac{1}{4} = .25$$

Since  $\epsilon < 1$ , price is inelastic  
 a 4% increase in price will lead to a  
1% decrease in demand.

- (e) Find the revenue when price is \$3 and when it is \$3.10. Did revenue increase or decrease by increasing price? How could we have known this by just looking at elasticity of demand?

$$\begin{aligned} \text{Revenue} &= \text{price} \cdot (\text{units sold}) \\ &= \text{price} \cdot (-2p + 10) \\ @ \$3 \text{ Revenue} &= 3(-2(3) + 10) = 12 \\ @ \$3.10 \text{ Revenue} &= 3.10(-2(3.10) + 10) = 11.78 \end{aligned}$$

Revenue decreased, expected since  $\epsilon > 1$

- (f) Find the revenue when price is \$1 and when it is \$1.10. Did revenue increase or decrease by increasing price? How could we have known this by just looking at elasticity of demand?

$$\begin{aligned} @ \$1 \text{ Revenue} &= 1(-2(1) + 10) = 8 \\ @ \$1.10 \text{ Revenue} &= 1.1(-2(1.1) + 10) = 8.58 \end{aligned}$$

Revenue increased, expected since  $\epsilon < 1$

- (g) What price maximizes revenue? Answer both using elasticity of demand and again using the fact that we have a quadratic revenue and compare your answers.

① Revenue maximized when  $\epsilon = 1$

$$\epsilon = \frac{2p}{10-2p} = 1$$

$$2p = 10 - 2p$$

$$10 = 4p$$

$$p = \frac{10}{4} = \text{\$}2.5$$

Revenue is

$$\begin{aligned} p(-2p + 10) \\ = -2p^2 + 10p \end{aligned}$$

$$\text{max @ } p = \frac{-b}{2a}$$

$$= \frac{-10}{2(-2)} = \frac{10}{4}$$

← same →  $\text{\$}2.5$

2. Jennifer finds the demand for her custom socks is given by  $x = 1/p^3$ .



(a) Find elasticity of demand.

$$\epsilon = - \frac{dx}{dp} \cdot \frac{p}{x}$$

$$= \frac{3}{p^4} \cdot \frac{p}{1/p^3} = \frac{3}{p^4} \cdot p \cdot \frac{p^3}{1} = 3$$

$$\begin{aligned} x &= 1/p^3 = p^{-3} \\ \frac{dx}{dp} &= -3p^{-4} \\ &= -3/p^4 \end{aligned}$$

(b) Give an interpretation for elasticity of demand when price is \$1.

@  $p = \$1$ ,  $\epsilon = 3/1$  which means a  
1% increase in price leads to a  
3% decrease in demand  
 (true for all values of  $p$ )

(c) What price will maximize ~~profit~~ <sup>Revenue</sup>? (Use elasticity to find your answer).

Since  $\epsilon = 3$  for all  $p$  and  $\epsilon = 3 > 1$   
 that means revenue is always decreasing.  
 So she should sell socks at the lowest  
 possible price ( $\$.01$ ?) to maximize  
 revenue.

More Practice from Textbook 4.5: You should do as many problems from each set (1-12, 13-22, 23-27), as needed until you are comfortable with these techniques.