

## Section 4.4: Derivatives of Exponential and Logarithmic Functions

### Section Objectives:

- Apply the chain rule to take derivatives of more complicated functions involving logarithms and exponentials.
- Use properties of logarithms to find  $\frac{d}{dx}(a^x)$  and then remember that general derivative rule.
- Use properties of logarithms to find  $\frac{d}{dx}(\log_a |x|)$  and then remember that general derivative rule.

### Practice Problems

Practice, Practice, Practice!

1. Find the derivative of  $f(x) = e^{3x^2+3x+1}$

$$e^{(3x^2+3x+1)} (6x+3)$$

2. Find the derivative of  $f(x) = \ln(3x + e^x)$

$$\frac{1}{3x+e^x} \cdot (3+e^x)$$

3. Find the derivative of  $f(x) = \frac{x}{e^{3x^2}+2}$  quotient rule

$$\frac{(e^{3x^2}+2)(1) - x(e^{3x^2} \cdot 6x)}{(e^{3x^2}+2)^2}$$

4. Find the derivative of  $f(x) = e^{x^2}(e^x)^2$  product rule

$$\underbrace{(e^{x^2})}_{f'} \cdot 2 \cdot \underbrace{(e^x)}_{g'} \cdot (e^x) + \underbrace{e^{x^2}}_{f'} \cdot (2x) \cdot \underbrace{(e^x)^2}_{g'}$$

5. Find the derivative of  $f(x) = \ln(x^7) + \ln(x)^7$

$$\frac{1}{x^7} \cdot 7x^6 + 7(\ln(x))^6 \left(\frac{1}{x}\right) = \frac{7}{x} + \frac{7(\ln(x))^6}{x}$$

6. Find the derivative of  $f(x) = \ln(\sqrt{e^x+2}+1)$

$$\frac{1}{\sqrt{e^x+2}+1} \cdot \frac{1}{2}(e^x+2)^{-1/2} \cdot (e^x) = \frac{e^x}{2(\sqrt{e^x+2}+1)\sqrt{e^x+2}}$$

7. Let  $f(x) = 3^x$ .

(a) Rewrite this function as  $f(x) = e^{\text{something}}$ .

$$f(x) = 3^x = e^{\ln(3^x)} = e^{x \cdot \ln(3)}$$

(b) Use the chain rule to take the derivative of the function above.

$$\frac{d}{dx} \left( e^{\ln(3)x} \right) = \underbrace{e^{\ln(3)x}}_{= 3^x} \cdot \ln(3) = \boxed{3^x \cdot \ln(3)}$$

(c) Use this technique to find the derivative of the general function  $f(x) = a^x$ .

$$\frac{d}{dx} (a^x) = \frac{d}{dx} \left( e^{\ln(a^x)} \right) = \frac{d}{dx} \left( e^{x \ln(a)} \right) = e^{x \ln(a)} \cdot \ln(a) = \boxed{a^x \cdot \ln(a)}$$

8. Let  $f(x) = \log_3(x)$ .

(a) Rewrite this function using only  $\ln(\text{something})$ , maybe more than once. Hint: Change of base formula.

$$\log_3 x = \frac{\ln(x)}{\ln(3)} = \frac{1}{\underbrace{\ln(3)}_{\text{constant}}} \cdot \ln(x)$$

(b) Use the chain rule to take the derivative of the function above.

$$\frac{d}{dx} \left( \frac{1}{\ln(3)} \ln x \right) = \frac{1}{\ln(3)} \cdot \frac{1}{x} = \boxed{\frac{1}{\ln(3) \cdot x}}$$

(c) Use this technique to find the derivative of the general function  $f(x) = \log_a(x)$ .

$$\frac{d}{dx} \left( \log_a x \right) = \frac{d}{dx} \left( \frac{\ln x}{\ln a} \right) = \frac{1}{\ln(a)} \cdot \frac{1}{x} = \boxed{\frac{1}{x \ln(a)}}$$

**More Practice from Textbook 4.4:** You should do as many problems from each set (1-50, 51-62, 63-70, 71-76, 85-91), as needed until you are comfortable with these techniques. 63-70 are good practice for application problems.