Section 4.3: The Chain Rule
Section Objectives:

- Know how to use the chain rule to find the derivative of functions which are compositions.

Practice Problems
Practice, Practice, Practice!

1. Find the derivative of $f(x) = (3x+2)^3$.

$$3(3x+2)^2(3)$$

2. Find the derivative of $f(x) = e^{5x^2+2}$.

$$e^{5x^2+2}(10x)$$

3. Find the derivative of $f(x) = \sqrt{e^{4x}}$. $= (e^{4x})^{1/2} =$ or e^{2x}

so $f'(x) = \boxed{2e^{2x}}$

note : $\sqrt{e^{4x}} = e^{2x}$

$$= \frac{1}{2}(e^{4x})^{-1/2}\frac{d}{dx}(e^{4x})$$

$$= \frac{1}{2\sqrt{e^{4x}}} \cdot e^{4x} \cdot 4 = \boxed{2\sqrt{e^{4x}}}$$

4. Find the derivative of $f(x) = e^x(\ln(x) + 7x^2)$.

product rule

$$\underset{f}{e^x}\underset{g'}{\left(\frac{1}{x} + 14x\right)} + \underset{f'}{e^x}\underset{g}{\left(\ln(x) + 7x^2\right)}$$

5. Find the derivative of $f(x) = \sqrt{\sqrt{x^3+1}+1}$. $= \left(\sqrt{x^3+1} + 1\right)^{1/2}$

$$f'(x) = \frac{1}{2}\left((x^3+1)^{1/2} + 1\right)^{-1/2} \cdot \frac{d}{dx}\left((x^3+1)^{1/2} + 1\right)$$

$$= \frac{1}{2}\left((x^3+1)^{1/2} + 1\right)^{-1/2}\left(\frac{1}{2}(x^3+1)^{-1/2}(3x^2)\right)$$

$$= \frac{3x^2}{4\left(\sqrt{\sqrt{x^3+1} + 1}\right)\left(\sqrt{x^3+1}\right)}$$

6. Find the derivative of $f(x) = \frac{e^x+2}{x^3+2x+\ln(x)}$.

quotient rule

$$\frac{(x^3+2x+\ln x)(e^x) - (e^x+2)(3x^2+2+\frac{1}{x})}{(x^3+2x+\ln x)^2}$$

7. The demand for a product is given by

$$p(x) = \frac{3000}{\ln(x^2+1)},$$

where x is the number of items sold in thousands and p is the price, per 1000 sold in dollars.

(a) Find and interpret $p(12)$ and $p'(12)$. What are the units?

$$P(12) = \frac{3600}{\ln(12^2+1)} = \$600.82$$

when 12,000 items are sold the price is \$600.82 per thousand.

$$P(x) = 3000\left(\ln(x^2+1)\right)^{-1}$$

$$P'(x) = -3000\left(\ln(x^2+1)\right)^{-2} \cdot \frac{d}{dx}\left(\ln(x^2+1)\right)$$

$$= -3000\left(\ln(x^2+1)\right)^{-2} \cdot \frac{1}{x^2+1} \cdot 2x$$

$$P'(x) = \frac{-6000x}{\ln(x^2+1)^2\,(x^2+1)}$$

$$P'(12) = \frac{-6000(12)}{(\ln(145))^2\,(145)} = \$-20.05/\text{thousand units}$$

when 12000 units are sold price must decrease at rate of -20.05 per thousand units sold

(b) Find the revenue function $R(x)$. Then find $R(12)$ and $R'(12)$. Give an interpretation and find the units.

$$R(x) = x \cdot p(x) = \frac{3000x}{\ln(x^2+1)} \qquad R(12) = \$7233.66$$

When 12,000 units are sold the revenue is \$7233.66. (units: dollars)

$$R'(x) = \frac{\ln(x^2+1)(3000) - (3600x)\frac{1}{x^2+1}(2x)}{\ln(x^2+1)}$$

$$R'(12) = \$1802.70$$

when 12,000 units sold revenue is increasing at rate of \$1802.70 per addition 1000 unit sold.

to sell additional 1000 units.

units: $\frac{\$ \text{ per thousand units}}{\text{thousand units}}$

More Practice from Textbook 4.3: You should do as many problems from each set (1-36, 37-44, 45-48), as needed until you are comfortable with these techniques. 37-44 are good practice for application problems.

Units: dollars / 1000 units.