Section 4.2
Section Objectives:

- Know the following derivative rules. Be comfortable using them in a variety of problems. (Only the bottom 2 are new).

function	derivative
$f(x) = c$	$f'(x) = 0$
$f(x) = x^n$	$f'(x) = nx^n - 1$
$f(x) = e^x$	$f'(x) = e^x$
$g(x) = c \cdot f(x)$	$g'(x) = c \cdot f'(x)$
$f(x) = \ln(x)$	$f'(x) = \frac{1}{x}$
$h(x) = f(x) \pm g(x)$	$h'(x) = f'(x) \pm g'(x)$
$f(x) = g(x)h(x)$	$f'(x) = g(x)h'(x) + g'(x)h(x)$
$f(x) = \frac{g(x)}{h(x)}$	$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{(h(x))^2}$

- Know the definitions and interpretations of average cost and marginal average cost.

Practice Problems

1. Evaluate the following derivatives using the rules we learned in this section. (Practice, practice practice!!)

   (a) $\frac{d}{dx}(e^x(x^2 - 2x))$ product rule

$$e^x\left(\frac{d}{dx}(x^2-2x)\right) + \left[\frac{d}{dx}(e^x)\right](x^2-2x)$$

$$= e^x(2x-2) + e^x(x^2-2x)$$

$$= e^x(\cancel{2x} - 2 + x^2 - \cancel{2x}) = \boxed{e^x(x^2-2)}$$

(b) $\frac{d}{dx}((\ln(x)+2)(x^5-\sqrt{x}))$

$$(\ln(x)+2)\left(\frac{d}{dx}(x^5-x^{1/2})\right)+\left[\frac{d}{dx}(\ln(x)+2)\right](x^5-\sqrt{x})$$

$$=\left(\ln(x)+2\right)\left(5x^4-\frac{1}{2}x^{-1/2}\right)+\left(\frac{1}{x}\right)\left(x^5-\sqrt{x}\right).$$

(c) $\frac{d}{dx}(x^3e^x\ln(x))$

$$=(x^3e^x)\frac{d}{dx}(\ln x)+\left[\frac{d}{dx}(x^3e^x)\right]\ln x$$

↑ another product rule

$$=x^3e^x\cdot\frac{1}{x}+(x^3e^x+3x^2e^x)\ln x$$

(d) $\frac{d}{dx}\left(\frac{x}{x+3}\right)$, quotient rule.

$$=\frac{(x+3)(1)-x(1)}{(x+3)^2}=\frac{3}{(x+3)^2}$$

(e) $\frac{d}{dx}\left(\frac{e^x}{\ln(x)+2}\right)$

$$=\frac{(\ln(x)+2)(e^x)-e^x\left(\frac{1}{x}\right)}{(\ln(x)+2)^2}$$

(f) $\frac{d}{dx}\left(\frac{\sqrt{x}+\ln(x)}{xe^x}\right)$

product rule ↓

$$=\frac{(xe^x)\left(\frac{1}{2}x^{-1/2}+\frac{1}{x}\right)-(\sqrt{x}+\ln(x))\left(xe^x+e^x\right)}{(xe^x)^2}$$

2. The demand equation for a certain product is given by $p = \frac{x+1}{1+x^2}$, where x is the number sold and p is the price. Find the marginal revenue functions. Then find $R'(10)$. Give an interpretation of your answer.

$R(x) = p \cdot x = \frac{x^2+x}{1+x^2}$

$R'(x) = \frac{(1+x^2)(2x+1) - (x^2+x)(2x)}{(1+x^2)^2}$

$= \frac{2x+1+2x^3+x^2-2x^3-2x^2}{(1+x^2)^2} = \frac{-x^2+2x+1}{(1+x^2)^2}$

$R'(10) = \frac{-100+20+1}{(101)^2}$

$= -.0077$

When we sell 10 products, the revenue is decreasing at rate of .77 cents per additional unit sold.

3. Derek is setting up a shop to sell gumball machines. His costs are given by $C(x) = 3x^2 + ln(x) - 1$ for $x > 2$ where x is hundreds of gumball machines ~~sold~~ made.

(a) Find and interpret his marginal cost at $x = 3$.

Marginal cost: $C'(x)$

$C'(x) = 6x + \frac{1}{x}$

$C'(3) = 18 + \frac{1}{3} = 18.\overline{3}$

After making 300 gumball machines is cost to make more is $18.33 per hundred gumball machines.

(b) Find and interpret his average cost at $x = 3$.

Average Cost $\overline{C}(x) = \frac{C(x)}{x}$

$\overline{C}(x) = 3x + \frac{\ln x}{x} - \frac{1}{x}$

$\overline{C}(3) = 9 + \frac{\ln 3}{3} - \frac{1}{3} = 9.03$

When creating 3 hundred gumball machines, the average cost is $9.03 per hundred machines.

(c) Find and interpret his marginal average cost at $x = 3$.

Marginal average cost $\frac{d}{dx}\left(\overline{C}(x)\right) = \frac{d}{dx}\left(\frac{C(x)}{x}\right)$.

$\overline{C}(x) = 3x + \frac{\ln x}{x} - \frac{1}{x}$ (x^{-1})

$\overline{C}'(x) = 3 + \left[\frac{x(\frac{1}{x}) - \ln(x)}{x^2}\right] + \frac{1}{x^2} = 3 + \frac{1-\ln x}{x^2} + \frac{1}{x^2} = \frac{3x^2-\ln x+2}{x^2}$

$\overline{C}'(3) = \frac{27-\ln(3)+2}{(9)} = 3.100$

More Practice from Textbook 4.2: You should do as many problems from each set (1-8, 9-12, 13-24, 25-32, 33-44, 45-50 (skip 51-58 and 59-61), as needed until you are comfortable with these techniques. 33-44 are ~~good practice for~~ application problems.

After creating 300 gumball machines, the average cost per item is increasing at a rate of $3.10 per 100 gumball machines.