Section 4.1
Section Objectives:

- Know the following derivative rules. Be comfortable using them in a variety of problems.

function	derivative
$f(x) = c$	$f'(x) = 0$
$f(x) = x^n$	$f'(x) = nx^n - 1$
$f(x) = e^x$	$f'(x) = e^x$
$g(x) = c \cdot f(x)$	$g'(x) = c \cdot f'(x)$
$f(x) = \ln(x)$	$f'(x) = \frac{1}{x}$
$h(x) = f(x) \pm g(x)$	$h'(x) = f'(x) \pm g'(x)$

- Solve problems involving derivatives, like finding where there is a vertical tangent.
- Know how to find and interpret derivatives in application problems.
- Know the definitions and interpretations of marginal cost, marginal revenue and marginal profit.

Practice Problems

1. Evaluate the following derivatives using the rules we learned in this section. (Practice, practice practice!!)

   (a) $\frac{d}{dx}(3x^2 + 2)$ $= 6x$

   (b) $\frac{d}{dx}(\sqrt{x} + 5x^3 + e^x)$ $= \frac{d}{dx}\left(x^{1/2} + 5x^3 + e^x\right) = \frac{1}{2}x^{-1/2} + 15x^2 + e^x$

   (c) $\frac{d}{dx}(e^x - 3\ln(x))$ $= e^x - 3/x$

   (d) $\frac{d}{dx}((x-1)^3)$ $= \frac{d}{dx}\left(x^3 - 3x^2 + 3x - 1\right) = 3x^2 - 6x + 3$

   (e) $\frac{d}{dx}\left(\frac{2x^3 + x + x^2e^x}{x^2}\right)$ $= \frac{d}{dx}\left(2x + x^{-1} + e^x\right) = 2 - x^{-2} + e^x$

   (f) $\frac{d}{dx}\left(\sqrt[4]{x^3} + \ln(x^2)\right)$ $= \frac{d}{dx}\left(x^{3/4} + 2\ln x\right) = \frac{3}{4}x^{-1/4} + 2/x$

2. Let $f(x) = x^4 + x^3$. Find all point where there are ~~vertical~~ horizontal tangents.

WANT $f'(x) = 0$

$f'(x) = 4x^3 + 3x^2 \Rightarrow 4x^3 + 3x^2 = 0$

$x^2(4x+3) = 0$

$\Rightarrow x = 0$ or $4x+3 = 0$

$\boxed{x = 0,\ -3/4}$

3. Let $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 9x + 5$. Find $x > 0$ where the slope of the tangent line is equal to 1. Then find the equation of the tangent line at that point.

WANT $f'(x) = 1$. $f'(x) = x^2 + 3x - 9$.

$x^2 + 3x - 9 = 1$

$\Rightarrow x^2 + 3x - 10 = 0$

$(x-2)(x+5) = 0$

$x = 2,\ x = -5$ NO

(want $x > 0$)

$x = 2$

Now find equation of line

pt: $(2, f(2)) = (2, 8/3 + 6 - 18 + 5)$

$= (2, -13/3)$

Slope: 1

$y - (-13/3) = 1(x-2) \Rightarrow y = x - 2 - \frac{13}{3}$

$y = x - 19/3$

4. David has decided to start a business that sells goldfish. He finds his revenue model to be $R(x) = \sqrt{x}(x + 7)$ where x is in hundreds of goldfish sold.

(a) Find $R'(x)$.

$R(x) = x^{1/2}(x+7) = x^{3/2} + 7x^{1/2}$

$R'(x) = \frac{3}{2}x^{1/2} + \frac{7}{2}x^{-1/2} = \frac{3\sqrt{x}}{2} + \frac{7}{2\sqrt{x}}$

(b) Find and interpret $R(4)$. Be sure to include units.

$R(4) = \sqrt{4}(4+7) = 2(11) = 22.$

If he sells 400 goldfish, he will make \$22.

(c) Find and interpret $R'(4)$. Be sure to include units.

$R'(4) = \frac{3\sqrt{4}}{2} + \frac{7}{2\sqrt{4}} = \frac{6}{2} + \frac{7}{4} = \frac{19}{4} = \$4.75$ / hundred goldfish

After he sells 400 goldfish, selling additional goldfish will cause his revenue to increase at a rate of \$4.75 per add'l 100 goldfish sold.

~~**More Practice from Textbook 1.1:** You should do as many problems from each set (1-6, 7, 8, 9-10, 11-24, 25-31, 32-38, 39-42, 43-50, 51-58, 59-66, 67-88) as needed until you are comfortable with these techniques. 67-88 are good practice for more complicated application problems.~~