Section 3.4: Local Linearity
Section Objectives:

- Find the equation of the tangent line to a function and use it to approximate the function.
- Interpret marginal cost, marginal revenue and marginal profit in terms of linear approximations.

Practice Problems

1. Let $f(x) = \sqrt{x}$. $= x^{1/2}$

   (a) Find the equation of the tangent line at $x = 4$.

   $f(4) = \sqrt{4} = 2$

   point $(4, 2)$ slope: $\frac{1}{4}$

   $f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

   $y - 2 = \frac{1}{4}(x - 4)$

   $f'(4) = \frac{1}{4}$

   $\ell(x) = y = \underbrace{\frac{1}{4}(x-4) + 2}_{}$

   tangent line

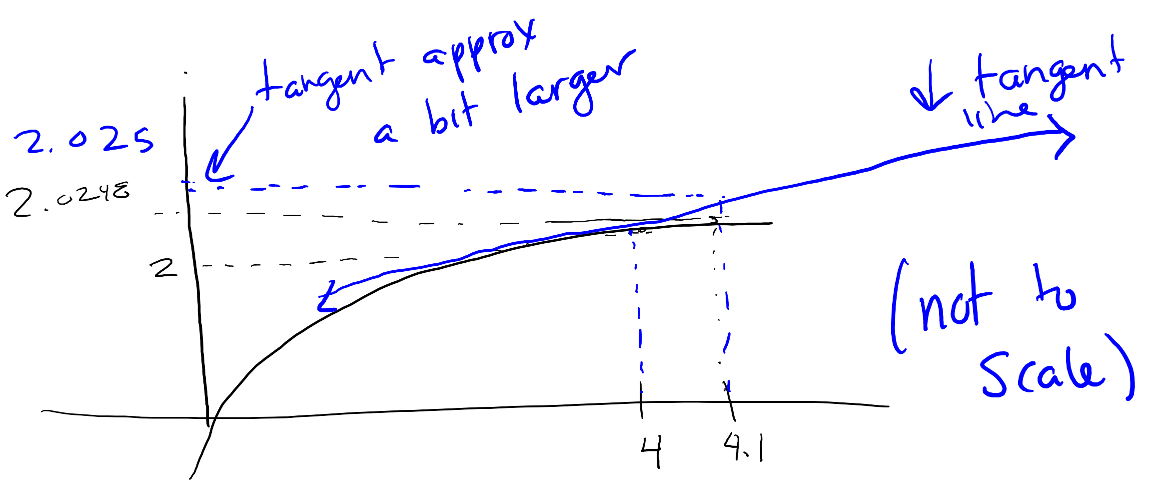
   easier to leave factored like this.

   (b) Use the equation of the tangent line to approximate $\sqrt{4.1}$.

   $$\sqrt{4.1} = f(4.1) \approx \ell(4.1) = \frac{1}{4}(4 - 4.1) + 2$$
   $$= \frac{1}{4}(.1) + 2$$
   $$= .025 + 2$$
   $$= 2.025$$

   (c) Use a calculator to find the exact value of $\sqrt{4.1}$. Compare it to your estimate. Illustrate with a graph.

   exact value: 2.0248, close to 2.025



2. Find the equation of the tangent line to $f(x) = \ln(x)$ at $x = 1$. Use this to approximate $\ln(2)$ and $\ln(1/2)$. Compare to the actual values. Illustrate with a sketch of the graphs.

$f(1) = \ln(1) = 0$ pt: $(1, 0)$

$f'(x) = \frac{1}{x}$ $f'(1) = 1$ slope: 1

eqn:

$y - 0 = 1(x - 1)$

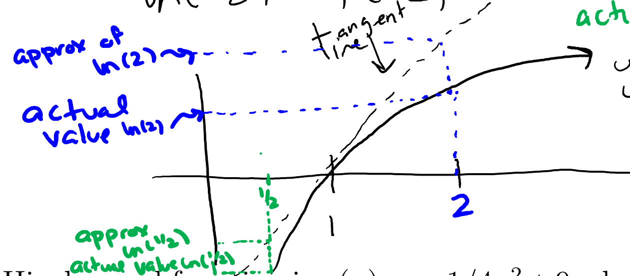
$l(x) = y = x - 1$

$\ln(2) = f(2) \approx l(2) = 2 - 1 = 1$

ln(2) actual value is .693

$\ln(1/2) = f(1/2) \approx l(1/2) = 1/2 - 1 = -\frac{1}{2}$

actual value $-.693$



3. Derek is selling homemade cookies. His demand function is $p(x) = -1/4x^2 + 9$ where x is boxes of cookies sold and p is price per box of cookies, in dollars. Find and interpret $p(3)$ and $p'(3)$. Use these to approximate $p(4)$. How does this compare to the actual value?

$p(3) = -\frac{1}{4}(9) + 9 = \frac{-9}{4} + 9 = \$6.75$

If he wants to sell 3 boxes, the price should be \$6.75 per box.

$p'(x) = -\frac{1}{2}x$ so $p'(3) = -\frac{3}{2} = -1.5 \; \frac{\$\text{s per box}}{\text{box}}$

After selling 3 boxes, the price must decrease at a rate of \$1.50 per box to sell add'l boxes.

4. Faith is selling computer monitors. She finds from her revenue function that $R(30) = 4000$ and that $R'(30) = 5$. Approximately how much more revenue will she get between selling the 30th monitor and the 31st monitor?

$R'(30) = 5$ means that after selling 30 monitors, her revenue is increasing at a rate of \$5 per additional monitor sold. So we will get approximately \$5 more for 31st monitor.

5. Bella makes and sells custom UConn hoodies. She has \$500 of startup costs, it costs \$10 of material to make each hoodie and she sells them for \$15.

   (a) What are her cost, revenue and profit functions?

$C(x) = 500 + 10x$

$R(x) = 15x$

$P(x) = 15x - (500 + 10x)$

$= 5x - 500.$

   (b) Find and interpret $C'(30)$, $R'(30)$ and $P'(30)$, the marginal cost, marginal revenue and marginal profit?

$C'(x) = 10$ so $C'(30) = 10$, after making 30 hoodies the cost for each additional is \$10.

$R'(x) = 15$ so $R'(30) = 10$ after making 30 hoodies the revenue for each additional is \$15.

$P'(x) = 5$ so $P'(30) = 5$. After selling 30 hoodies her profits is \$5 per add'l hoodie.

   (c) What do you notice about $C'(x)$, $R'(x)$ and $P'(x)$ for different x values? Why is this true?

$C'(x)$, $R'(x)$ & $P'(x)$ are all constant - they don't depend on x. This is because C, R & P are linear functions so their slopes are constant.

Thus increase in cost, revenue & profit don't depend on # of items sold.

More Practice from Textbook 4.4: You should do as many problems from each set (1-50, 51-62, 63-70, 71-76, 85-91), as needed until you are comfortable with these techniques.63-70 are good practice for application problems.

Note: this means its pretty easy to answer these questions <u>without</u> derivatives. For more complex C, R & P, we can really see the power of derivatives.