Section 3.3
Section Objectives:

- Know the connection between instantaneous rate of change and the derivative.
- Know the limit definition of $f'(x)$.
- Be able to use the limit definition of the derivative to find the derivative of linear function, quadratics, cubics, 1/linear functions and $\sqrt{\text{linear functions}}$.
- Know what makes a function not be differentiable at a point (discontinuities, corners/cusps and vertical tangents).
- Be able to sketch the graph of the derivative of a function given a graph of the function.
- Know how to find the units of the derivative function and how to interpret the derivative.

Practice Problems

1. Use the limit definition of the derivative the find $f'(x)$ for each of the following functions. Then find $f'(1)$ and the equation for the tangent line at $x = 1$ and sketch a graph of $f(x)$ and the tangent line to $f(x)$ at $x = 1$.

   (a) $f(x) = 3x + 2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)+2-(3x+2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x}+3h+\cancel{2}-\cancel{3x}-\cancel{2}}{h} = \lim_{h\to 0} \frac{3h}{h} = \boxed{3}$$

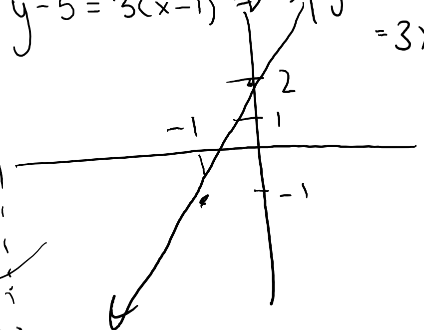
$$\boxed{f'(x) = 3}$$

(for linear function: tangent line & function are same)

$f'(1) = 3 \leftarrow$ slope

point $(1, f(1))$
$= (1, 5)$

$y - 5 = 3(x-1) \Rightarrow y = 3x - 3 + 5$
$= 3x + 2$



   (b) $f(x) = x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h} = \lim_{h\to 0} \frac{2x\cancel{h}+h^{\cancel{2}}}{\cancel{h}}$$

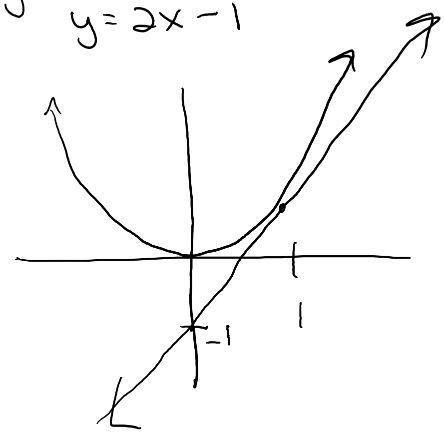
$$= \lim_{h\to 0} 2x + h = 2x$$

$$\boxed{f'(x) = 2x}$$

$f'(1) = 2(1) = 2.$

point $(1,1)$

$y - 1 = 2(x-1)$
$y = 2x - 1$



(c) $f(x) = x^3$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}^2} - \cancel{x^3}}{\cancel{h}}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 = 3x^2.$$

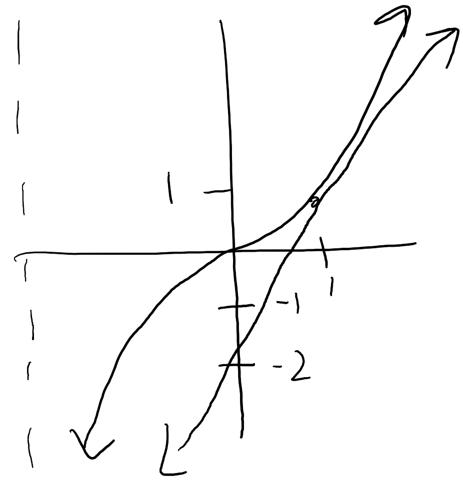
$$f'(x) = 3x^2$$

$$f'(1) = 3(1)^2 = 3$$

point: $(1,1)$.

$$y - 1 = 3(x-1)$$

$$y = 3x - 2$$



(d) $f(x) = \dfrac{1}{x+2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x+2}{(x+h+2)(x+2)} - \frac{(x+h+2)}{(x+h+2)(x+2)}}{h} = \lim_{h\to 0} \frac{\cancel{x} + \cancel{2} - \cancel{x} - \cancel{h} - \cancel{2}}{\cancel{h}(x+h+2)(x+2)}$$

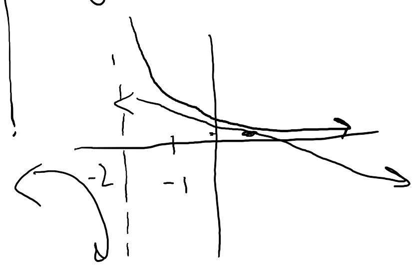
$$= \lim_{h\to 0} \frac{-1}{(x+h+2)(x+2)} = \frac{-1}{(x+2)^2}$$

$$f'(1) = \frac{-1}{3^2} = -\frac{1}{9}$$

$$f(1) = \frac{1}{3} \qquad (1, 1/3)$$

$$y - 1/3 = -\frac{1}{9}(x-1)$$

$$y = -\frac{1}{9}x + \frac{4}{9}$$



(e) $f(x) = \sqrt{x+3}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h}$$

multiply by conjugate

$$= \lim_{h\to 0} \frac{(\sqrt{x+h+3} - \sqrt{x+3})(\sqrt{x+h+3} + \sqrt{x+3})}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h\to 0} \frac{\cancel{x} + \cancel{h} + \cancel{3} - \cancel{x} - \cancel{3}}{\cancel{h}(\sqrt{x+h+3} + \sqrt{x+3})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}} = \boxed{\frac{1}{2\sqrt{x+3}}}$$

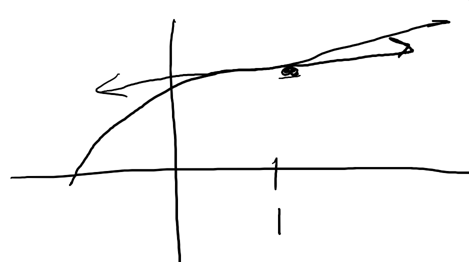
at $x = 1$

$$f'(1) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

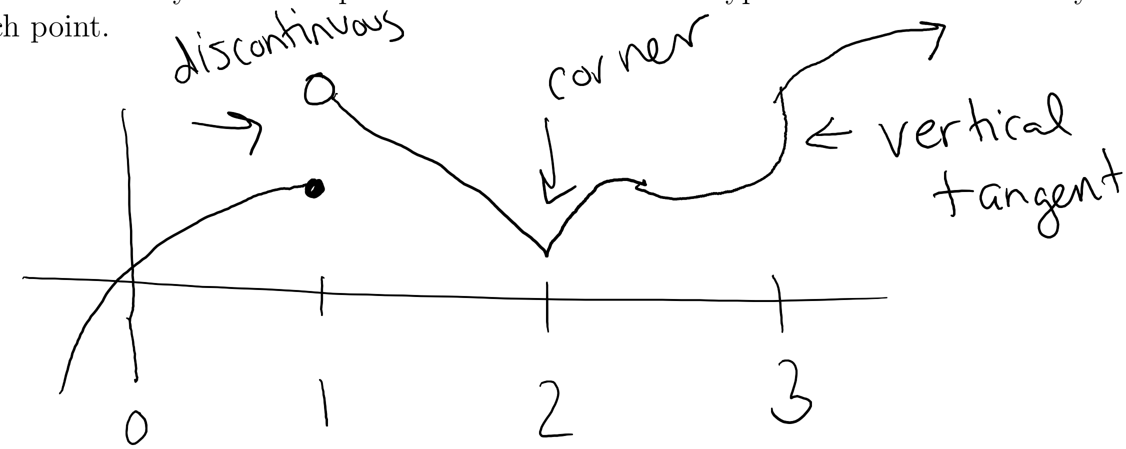
$$f(1) = 2.$$

$$y - 2 = \frac{1}{4}(x-1)$$

$$y = 1/4\, x + \frac{7}{4}$$



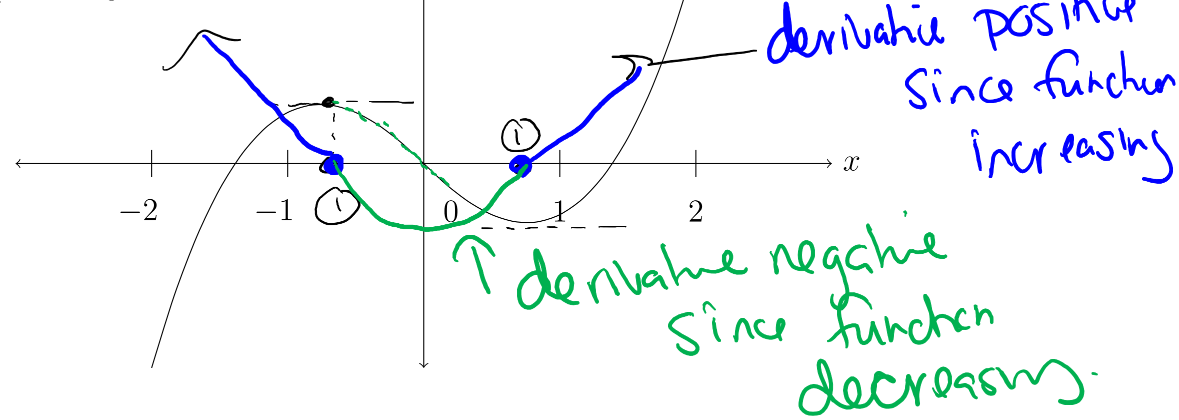
2. Sketch a graph of a function which is differential everywhere except $x = 1, 2$ and 3 and continuous everywhere except $x = 1$. Have a different type of non-differentiability at each point.



3. The graph of a function is given below. Use it to sketch the graph of its derivative on the same axis.

(1) Find where derivative is 0

(2) where is derivative >0 (increasing function)



4. The function $U(r)$ tells the number of umbrellas sold per day by the campus bookstore as a function of r, inches of rain. What are the units of $U(r)$ and $U'(r)$. Give an interpretation of $U(2) = 13$. Given an interpretation of $U'(3) = 2$.

$U(r)$ units : umbrellas per day

$U'(r)$ units : (umbrella per day) per inch.

$U(2) = 13$ if 2 inches of rain, sell 13 umbrellas per day

$U'(2) = 2$ if 2 inches of rain, for every add'l inch sell two more umbrellas per day.

More Practice from Textbook 1.1: You should do as many problems from each set (1-12, 13-16, 17-22, 23-24, 25-26, 27-32, 33-35, 36-40, 41-52, 53-56, 57-68), as needed until you are comfortable with these techniques. 41-52 are good practice for application problems.

or sell an additional 2 umbrellas per day per inch of rain.