Section 3.3
Section Objectives:

- Know the connection between instantaneous rate of change and the derivative.
- Know the limit definition of $f'(x)$.
- Be able to use the limit definition of the derivative to find the derivative of linear function, quadratics, cubics, 1/linear functions and $\sqrt{\text{linear functions}}$.
- Know what makes a function not be differentiable at a point (discontinuities, corners/cusps and vertical tangents).
- Be able to sketch the graph of the derivative of a function given a graph of the function.
- Know how to find the units of the derivative function and how to interpret the derivative.

Practice Problems

1. Use the limit definition of the derivative the find $f'(x)$ for each of the following functions. Then find $f'(1)$ and the equation for the tangent line at $x = 1$ and sketch a graph of $f(x)$ and the tangent line to $f(x)$ at $x = 1$.

   (a) $f(x) = 3x + 2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)+2-(3x+2)}{h}$$

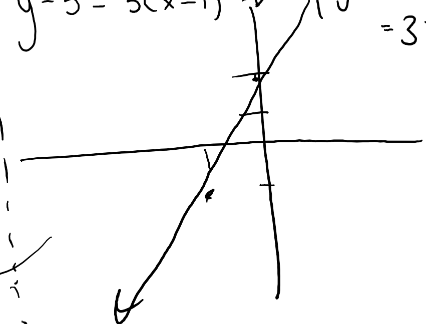
$$= \lim_{h\to 0} \frac{\cancel{3x} + 3h + \cancel{2} - \cancel{3x} - \cancel{2}}{h} = \lim_{h\to 0} \frac{3h}{h} = \boxed{3}$$

$f'(x) = 3$ (for linear function: tangent line & function are same)

$f'(1) = 3 \leftarrow$ slope

point $(1, f(1))$
$= (1, 5)$

$y - 5 = 3(x-1) \Rightarrow y = 3x - 3 + 5$
$= 3x + 2$



   (b) $f(x) = x^2$

$$f'(x) = \lim_{h\to 0} f(x+h) - f$$

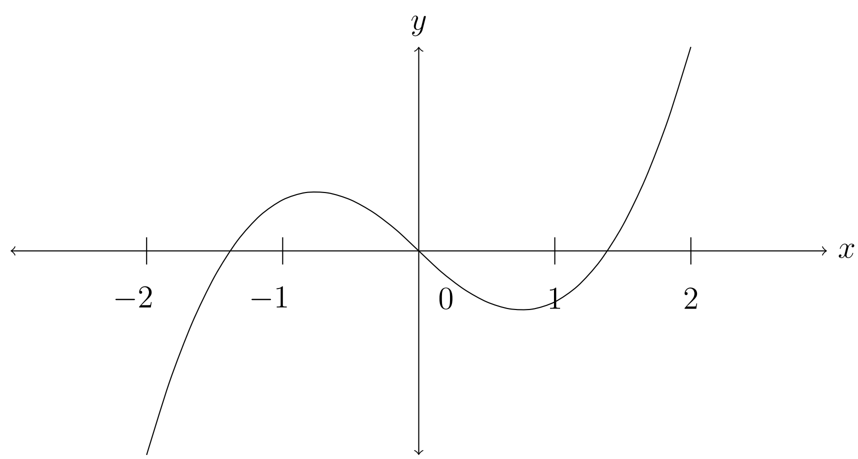
(c) $f(x) = x^3$

(d) $f(x) = \dfrac{1}{x+2}$

(e) $f(x) = \sqrt{x+3}$

2. Sketch a graph of a function which is differential everywhere except $x = 1, 2$ and 3 and continuous everywhere except $x = 1$. Have a different type of non-differentiability at each point.

3. The graph of a function is given below. Use it to sketch the graph of its derivative on the same axis.



4. The function $U(r)$ tells the number of umbrellas sold per day by the campus bookstore as a function of r, inches of rain. What are the units of $U(r)$ and $U'(r)$. Give an interpretation of $U(2) = 13$. Given an interpretation of $U'(3) = 2$.

More Practice from Textbook 1.1: You should do as many problems from each set (1-12, 13-16, 17-22, 23-24, 25-26, 27-32, 33-35, 36-40, 41-52, 53-56, 57-68), as needed until you are comfortable with these techniques. 41-52 are good practice for application problems.