Section 3.2
Section Objectives:

- Know how to find the average rate of change of a function and how it relates to the slope of the secant line.
- Know the connection between average rate of change and instantaneous rate of change and how to find instantaneous rate of change.
- Know the connection between instantaneous rate of change and the slope of the tangent line.
- Be able to find the equation of the tangent line to a function at a point.

Practice Problems

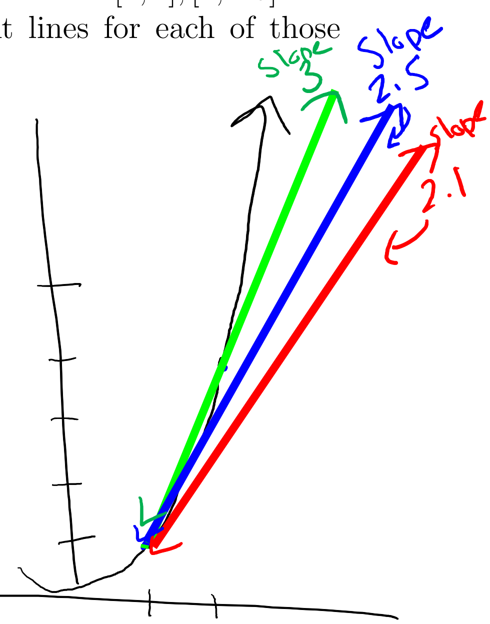
1. Let $f(x) = x^2$. Find the average rate of change over the intervals $[1,2], [1,1.5]$ and $[1,1.1]$. Sketch a graph of the function and draw in secant lines for each of those intervals. Label each secant line with its slope.

$f(x) = x^2$

$[1,2]$: $\dfrac{f(2)-f(1)}{1} = \dfrac{4-1}{1} = \boxed{3}$

$[1,1.5]$: $\dfrac{f(1.5)-f(1)}{1.5-1} = \dfrac{2.25-1}{.5} = \boxed{2.5}$

$[1,1.1]$: $\dfrac{f(1.1)-f(1)}{.1} = \dfrac{1.1^2-1}{.1} = \boxed{2.1}$



2. Let $f(x) = \ln(x)$. Find the average rate of change on the intervals $[2, 2+h]$ and $[2-h, 2]$ for several small values of h. Use this is estimate the instantaneous rate of change at $x = 2$.

Let $h = .1, .01, .001$, for $[a,b]$ calculate $\dfrac{f(b)-f(a)}{b-a}$

$[2, 2.1]$

$[$

3. Find the instantaneous rate of change of $f(x) = x^2 + 3x$ at $x = 3$, using algebraic techniques for limits. (You must use the limit definition for instantaneous rate of change.)

4. Find the instantaneous rate of change of $f(x) = \sqrt{x-1}$ at $x = 5$, using algebraic techniques for limits. (You must use the limit definition for instantaneous rate of change.) Hint: Multiply by the conjugate.

5. Find the equation for the tangent line to $f(x) = x^3 - 2x$ at $x = 1$. Graph both $f(x)$ and its tangent line, on the same graph.

More Practice from Textbook 3.2: You should do as many problems from each set (1-10, 11-12, 13-18, 19-20, 21-32, 33-36, 37-40, 41-70, 71-76), as needed until you are comfortable with these techniques. 41-70 are good practice for application problems.