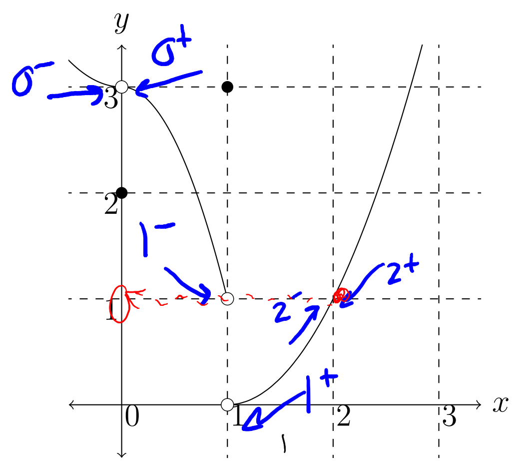
**Section 3.1**
**Section Objectives:**

- Understand the idea behind the definition of a limit.
- Estimate limits from graphs, a table of values and using a calculator.
- Understand and find one sided limits graphically and numerically.
- Know what makes a limit approach positive or negative infinity.
- Know the definition of a vertical asymptote and how to find them.
- Know the algebraic rules for limits including, multiplication by constants, addition and subtraction, multiplication and division and powers.
- Use algebraic techniques (factoring/multiplying by conjugate) to evaluate limits.
- Know the definition of continuity using limits.

**Practice Problems**

1. The graph of $y = f(x)$ is below. Use it to compute each limit or explain why it doesn't exist.




(a) $\lim_{x\to 0^-} f(x) = 3$

(b) $\lim_{x\to 1^-} f(x) = 1$

(c) $\lim_{x\to 2^-} f(x) = 1$

(d) $\lim_{x\to 0^+} f(x) = 3$

(e) $\lim_{x\to 1^+} f(x) = 0$

(f) $\lim_{x\to 2+} f(x) = 1$

(g) $\lim_{x\to 0} f(x) = 3$ since $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = 3$

(h) $\lim_{x\to 1} f(x)$ DNE since $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

(i) $\lim_{x\to 2} f(x) = 1 = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

(j) $f(0) = 2$ (closed dot)

(k) $f(1) = 3$ (closed dot)

(l) $f(2) =$ (continuous).

$\boxed{1}$ ← function value

2. Estimate $\lim_{x\to 0^+} \frac{\ln(x)}{x}$ using a calculator. Show your work and explain your reasoning.

| $x$ | .1 | .01 | .002 | .0007 |
|---|---|---|---|---|
| $\frac{\ln x}{x}$ | -23 | -461 | -3107 | -10378 |

As $x$ gets closer & closer to 0, approaching from right (positive values) $f(x) = \frac{\ln x}{x}$ goes to $\boxed{-\infty}$.

3. Where does the function $f(x) = \frac{x-3}{x+1}$ have a vertical asymptote. Explain your reasoning algebraically. $\lim_{x\to a} f(x) = \pm\infty$ when we divide by 0.

TRY $a=-1$ (makes denominator 0

$$\lim_{x\to -1} \frac{x-3}{x+1} = \frac{-4}{0} = \frac{\# \neq 0}{\text{zero}} = \pm\infty.$$

$\boxed{\text{Thus } x=-1 \text{ is vertical asymptote.}}$

4. Let $\lim_{x\to 7} f(x) = 2$, $\lim_{x\to 7} g(x) = 3$, $\lim_{x\to 7} h(x) = 4$. Find $\lim_{x\to 7}\left(\frac{g(x)}{f(x)} + \sqrt{h(x)} + 3x + 4\right)$.

$$\lim_{x\to 7}\left(\frac{g(x)}{f(x)} + \sqrt{h(x)} + 3x + 4\right) = \frac{\lim_{x\to 7} g(x)}{\lim_{x\to 7} f(x)} + \sqrt{\lim_{x\to 7} h(x)} + 3\lim_{x\to 7} x + 4$$

$$= \frac{3}{2} + \sqrt{4} + 3(7) + 4 = \boxed{\phantom{00}}$$

28.5

5. Evaluate the following limit using algebraic techniques. Check your work by using a calculator to estimate the limit. What values did you enter on the calculator?

(a) $\lim_{x\to 5} \frac{x^2-3x-10}{x-5}$ $= \frac{\text{"}0\text{"}}{0}$ (plug in) → try to factor

$$\lim_{x\to 5} \frac{\cancel{(x-5)}(x+2)}{\cancel{(x-5)}} = \lim_{x\to 5} x+2 = \boxed{7}$$

Try on calculator

| $x$ | 5.1 | 5.01 | 4.99 |
|---|---|---|---|
| $f(x)$ | 7.1 | 7.01 | 6.99 |

← also approaching 7

(b) $\lim_{x\to 2} \frac{\sqrt{11-x}-3}{x-2}$ $= \frac{\text{"}0\text{"}}{0}$, multiply by conjugate

$$\lim_{x\to 2} \frac{(\sqrt{11-x}-3)}{(x-2)}\frac{(\sqrt{11-x}+3)}{(\sqrt{11-x}+3)} = \lim_{x\to 2} \frac{11-x-9}{(x-2)(\sqrt{11-x}+3)} = \lim_{x\to 2} \frac{(2-x)}{(x-2)(\sqrt{11-x}+3)}$$

$$= \lim_{x\to 2} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{11-x}+3)} = \lim_{x\to 2} \frac{-1}{\sqrt{11-x}+3} = \frac{-1}{\sqrt{9}+3} = \boxed{\frac{-1}{6}}$$

| $x$ | 2.1 | 2.01 | 1.9 | 1.99 |
|---|---|---|---|---|
| $f(x)$ | -.1671 | -.1667 | -.1662 | -.1666 |

& approach $-1/6 = -.1666$.

6. Consider the function

$$f(x) = \begin{cases} x + k & x < 0 \\ m & x = 0 \\ kx + 3 & x > 0 \end{cases}.$$

(a) What is $\lim_{x \to 0^-} f(x)$? Your answer will depend on $k$.

when $x<0$, plug into top equation

$$\lim_{x \to 0^-} x+k = \boxed{k}$$

(b) What is $\lim_{x \to 0^+} f(x)$? Your answer will depend on $k$.

When $x>0$ plug into 3rd equation

$$\lim_{x \to 0^+} kx+3 = \boxed{3}$$

(c) Recall that $\lim_{x \to 0} f(x)$ only exists if $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$. What value of $k$ makes this true?

Want $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$$\Rightarrow \boxed{k = 3}$$

(d) The function $f(x)$ is only continuous if $\lim_{x \to 0} f(x) = f(0)$. What value of $m$ makes this true.

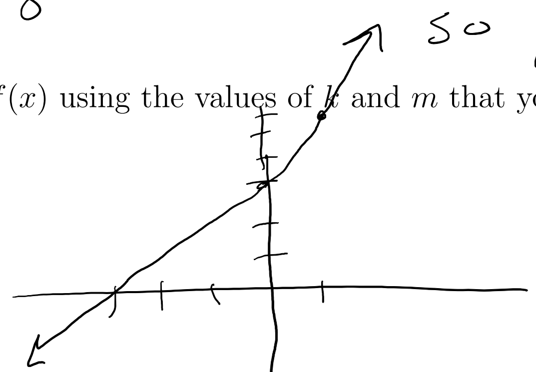
$f(0) = m$

Want $\lim_{x \to 0} f(x) = m$. Have $\lim_{x \to 0} f(x) = 3$

so $\boxed{m=3}$

(e) Sketch a graph of $f(x)$ using the values of $k$ and $m$ that you found.

$$f(x) = \begin{cases} x+3 & x<0 \\ 3 & x=0 \\ 3x+3 & x>0 \end{cases}$$



**More Practice from Textbook** 3.1: You should do as many problems from each set (1-6, 7-13, 14-25, 26-36, 37-46, 47-49, 50-60, 61-81), as needed until you are comfortable with these techniques. 48-65 are good practice for application problems.