

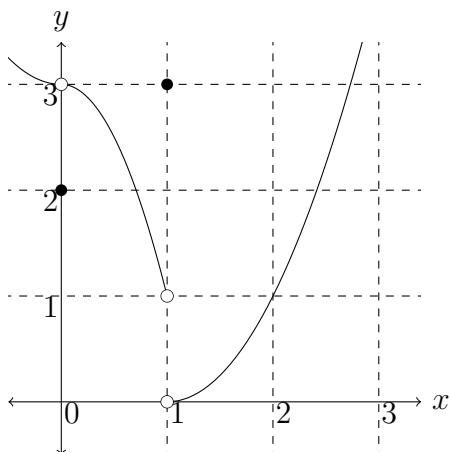
## Section 3.1

### Section Objectives:

- Understand the idea behind the definition of a limit.
- Estimate limits from graphs, a table of values and using a calculator.
- Understand and find one sided limits graphically and numerically.
- Know what makes a limit approach positive or negative infinity.
- Know the definition of a vertical asymptote and how to find them.
- Know the algebraic rules for limits including, multiplication by constants, addition and subtraction, multiplication and division and powers.
- Use algebraic techniques (factoring/multiplying by conjugate) to evaluate limits.
- Know the definition of continuity using limits.

### Practice Problems

1. The graph of  $y = f(x)$  is below. Use it to compute each limit or explain why it doesn't exist.



(a)  $\lim_{x \rightarrow 0^-} f(x)$

(g)  $\lim_{x \rightarrow 0} f(x)$

(b)  $\lim_{x \rightarrow 1^-} f(x)$

(h)  $\lim_{x \rightarrow 1} f(x)$

(c)  $\lim_{x \rightarrow 2^-} f(x)$

(i)  $\lim_{x \rightarrow 2} f(x)$

(d)  $\lim_{x \rightarrow 0^+} f(x)$

(j)  $f(0)$

(e)  $\lim_{x \rightarrow 1^+} f(x)$

(k)  $f(1)$

(f)  $\lim_{x \rightarrow 2^+} f(x)$

(l)  $f(2)$

2. Estimate  $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x}$  using a calculator. Show your work and explain your reasoning.

3. Where does the function  $f(x) = \frac{x-3}{x+1}$  have a vertical asymptote. Explain your reasoning algebraically.

4. Let  $\lim_{x \rightarrow 7} f(x) = 2$ ,  $\lim_{x \rightarrow 7} g(x) = 3$ ,  $\lim_{x \rightarrow 7} h(x) = 4$ . Find  $\lim_{x \rightarrow 7} \left( \frac{g(x)}{f(x)} + \sqrt{h(x)} + 3x + 4 \right)$ .

5. Evaluate the following limit using algebraic techniques. Check your work by using a calculator to estimate the limit. What values did you enter on the calculator?

(a)  $\lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{x - 5}$

(b)  $\lim_{x \rightarrow 2} \frac{\sqrt{11-x} - 3}{x - 2}$

6. Consider the function

$$f(x) = \begin{cases} x + k & x < 0 \\ m & x = 0 \\ kx + 3 & x > 0 \end{cases}.$$

(a) What is  $\lim_{x \rightarrow 0^-} f(x)$ ? Your answer will depend on  $k$ .

(b) What is  $\lim_{x \rightarrow 0^+} f(x)$ ? Your answer will depend on  $k$ .

(c) Recall that  $\lim_{x \rightarrow 0} f(x)$  only exists if  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$ . What value of  $k$  makes this true?

(d) The function  $f(x)$  is only continuous if  $\lim_{x \rightarrow 0} f(x) = f(0)$ . What value of  $m$  makes this true.

(e) Sketch a graph of  $f(x)$  using the values of  $k$  and  $m$  that you found.

**More Practice from Textbook 3.1:** You should do as many problems from each set (1-6, 7-13, 14-25, 26-36, 37-46, 47-49, 50-60, 61-81), as needed until you are comfortable with these techniques. 48-65 are good practice for application problems.