Section 1.5
Section Objectives:

- Know the definition of the logarithm function $\log_a(x)$ as the inverse of a^x.
- Know the definition of the natural logarithm function $\ln(x) = \log_e(x)$
- Know the cancelation properties of exponentials and logarithms.
- Know the log rules for multiplication, division and powers.
- Solve equations using logarithms.
- Rewrite exponential functions by changing the base, and use the change of base formula for logs.

Practice Problems

1. Solve for x. Give an exact answer and an answer rounded to 3 decimal places.

$$2\ln(x) = 4.$$

$\div 2 \qquad \div 2$

$e^{\ln(x)} = e^{2} \Rightarrow \boxed{x = e^2 = 7.389}$

2. Simplify $e^{\frac{1}{2}\ln(16)}$

must move power first, otherwise e & ln can't cancel.

$e^{1/2\ln(16)} = e^{\ln(16^{1/2})} = 16^{1/2} = \sqrt{16} = 4$

3. Solve for x. Give an exact answer and an answer rounded to 3 decimal places.

$$\ln(x+1) - 5 = \ln(2).$$

$\ln(x+1) + 5 = \ln(2)$, add 5 to both sides

$e^{\ln(x+1)} = e^{\ln(2)+5}$, e both sides

$x+1 = e^{\ln(2)} \cdot e^{5}$ split the sum

$\Rightarrow x+1 = 2e^5$

$x = 2e^5 - 1$

$\boxed{= 295.826}$

4. Simplify $\log_3(x) + \log_3(2x+1) - \log_3(y)$.

$+$ becomes multiplication inside, $-$ becomes division.

$\log_3\left(\frac{x(2x+1)}{y}\right)$

5. Rewrite $f(x) = 3^x$ as e^{kx}, for some k.

Since $e^{\ln(a)} = a$ we can apply $e^{\ln(f(x))}$

$$e^{\ln(3^x)} = e^{x \cdot \ln(3)} = \boxed{e^{\ln(3) \cdot x}}$$

log rule, bring x down

6. Rewrite $log_5(x)$ using the natural log function: $\ln(x)$. Then check your answer by evaluating each function at $x = 25$.

$$\log_5 x \underset{\text{formula}}{\overset{\text{Change of base}}{=}} \boxed{\frac{\ln x}{\ln 5}}$$

$\Rightarrow$

at $x = 25$

$\log_5 25 = 2$, since $5^2 = 25$

$\frac{\ln 25}{\ln 5} = 2$ (calculator)

7. If an account earn 2% interest compounded monthly, how long does it take for the amount of money in the account to double?

Start w/ \$P. Want \$2P

$$\frac{2P}{P} = \frac{P}{P}\left(1 + \frac{0.02}{12}\right)^{12t}$$

$$2 = \left(1 + \frac{.02}{12}\right)^{12t}$$

ln both sides, bring down power

$$\ln(2) = 12 \cdot t \cdot \ln\left(1 + \frac{.02}{12}\right)$$

$$t = \frac{\ln(2)}{12 \ln\left(1 + \frac{.02}{12}\right)} = \boxed{34.69 \text{ years}}$$

8. If an account earn 2% interest compounded ~~monthly~~ continuously, how long does it take for the amount of money in the account to double?

Start w/ P, Want $2P$

$$\frac{2P}{P} = \frac{Pe^{.02t}}{P}$$

$$2 = e^{.02t}$$ ln both sides

$$\ln(2) = .02t$$

$$t = \frac{\ln(2)}{.02} = \boxed{34.66 \text{ years}}$$

9. If a population is growing exponentially according to the function $P(t) = P_0 e^{kt}$ where P_0 is the initial population, k is some positive constant and t is time in years, find the time needed for the population to double. Your answer will depend on k.

Start w/ P_0, Want $2P_0$

$$\frac{2P_0}{P_0} = \frac{P_0 e^{kt}}{P_0}$$

$$2 = e^{kt}$$, ln both sides

$$\ln(2) = kt$$

$$\boxed{t = \frac{\ln 2}{k}}$$

More Practice from Textbook 1.5: You should do as many problems from each set (1-6, 7-16, 17-22, 23-28, 29-40, 41-45, 46-47, 48-65, 66-74), as needed until you are comfortable with these techniques. 48-65 are good practice for application problems.