Section 1.3
Section Objectives:

- Know how to calculate compound interest.
- Know what the nominal rate, the effective annual yield, and present value are and how to find them.
- Understand the connection between compounding n times per year and compounding continuously.
- Know how to calculate interest compounded continuously and also find the effective annual yield and present value in these situations.
- Work with exponential functions (both of growth and decay) in a variety of settings.
- Be able to identify the properties of a graph of an exponential function.
- Solve equations with exponents.

Practice Problems

Solutions (Warning: Reading through the solutions is not nearly as effective as working out the problems on your own and then comparing your answers with the solutions.)

1. Samuel puts \$3,500 into a bank account. How much money will he have after 3 years in each of the following situations.

   (a) the account earns 1.4% interest compounded annually.

   $$3500(1+.014)^3 = 3649.07$$

   (b) the account earns 1.4% interest compounded monthly.

   $$3500\left(1+\frac{.014}{12}\right)^{3\cdot 12} = 3650.64$$

   (c) the account earns 1.4% interest compounded weekly (you may assume there are exactly 52 weeks in a year).

   $$3500\left(1+\frac{.014}{52}\right)^{3\cdot 52} = 3650.13$$

(d) the account earns 1.4% interest compounded daily. (Assume no leap years).

$$3500\left(1+\frac{.014}{365}\right)^{3\cdot 365} = 3650.13$$

(e) the account earns 1.4% interest compounded once per hour.

$$365 \text{ days} \cdot 24 \frac{\text{hrs}}{\text{day}} = 8760 \text{ hrs/year}$$

$$3500\left(1+\frac{.014}{8760}\right)^{8760\cdot 3} = 3650.13$$

(f) the account earns 1.4% interest compounded continuously

$$3500 \cdot e^{(0.014)\cdot 3} = 3650.13$$

(g) Relate the answers you got above to the idea of compounding continuously.

As we increase the number of times compounded per year we approach the value for compounded continuously.

(an rounded to nearest cent get the same value).

2. Patrick wants to give his future first born child \$3,000 on his 18th birthday. He knows his first born will be born exactly 8 years from today (he's psychic). How much money does he need to but into a bank account earning 5% interest today if that account:

(a) is compounded monthly

Patrick wants \$3,000 in 18+8 = 26 years
Call the amount he puts in P

$$P\left(1+\frac{.05}{12}\right)^{12\cdot 26} = \$3,000 \text{ so } P = \frac{3000}{\left(1+\frac{.05}{12}\right)^{12\cdot 26}}$$

$$= \boxed{\$819.81}$$

(b) is compounded continuously

Want

$$P\cdot e^{.05(26)} = 3000$$

$$P = \frac{3000}{e^{.05(26)}} = \boxed{\$817.60}$$

3. An account has a nominal rate of 2.2%. What is the effective annual rate if it:

(a) is compounded weekly

After one year

$$P\cdot\left(1+\frac{.022}{52}\right)^{52} \quad P(\underbrace{1.02224}))$$

so effective annual is 2.224%

(b) is compounded continuously

after one year

$$P\cdot e^{.022} = P(\underbrace{1.022243})$$

also 2.224% effective annual

4. Researchers found that from 1800 to 1950, the population of Connecticut was growing approximately exponentially. In 1860, the population was 460 thousand. In 1920, it was 1,381 thousand. We fill find and work with a model for the population size.

(a) By dividing the two population sizes, find percentage ~~growth~~ change of the population between 1860 and 1920.

$$\frac{1381 \text{ thousand}}{460 \text{ thousand}} = 3.0022$$

$= 300\%$ increase

(b) This tells us the percent growth over 60 years. So we know that (1+annual growth)^{60}=percentage growth from above. Solve for the annual growth.

$$(1 + \text{annual growth})^{60} = 3.6022 \Rightarrow 1 + \text{annual growth} = 1.0186$$

$$1 + \text{annual growth} = \sqrt[60]{3.0222}$$

$$\text{annual growth} = .0186 = \boxed{1.86\%}$$

(c) Use the information above to find a population model for Connecticut where the input is years since 1860.

We know $r = .0186$

We have Population $= P(1+.0186)^t$, where t is years since 1860. 1860 represents $t=0$ so we have $460 = P(1.0186)^0 \Rightarrow P = 460$

thus population $= \boxed{460(1.0186)^t}$

(d) What does the model predict the population of Connecticut will be in 1950? How does this compare to the actual population in 1950? (Google the 1950 Connecticut population)

1950 is 90 years after 1860 so $t=90$.

$460(1.0186)^{90} = 2415.95$ thousand or (Wolfram Alpha)

2.416 million, actual population 2.007 million

Kinda close...

(e) What does the model predict the population of Connecticut will be in 2018? How does this compare to the actual population? Why do you think the values are so different?

2018 is 158 years after 1860 so $t=158$

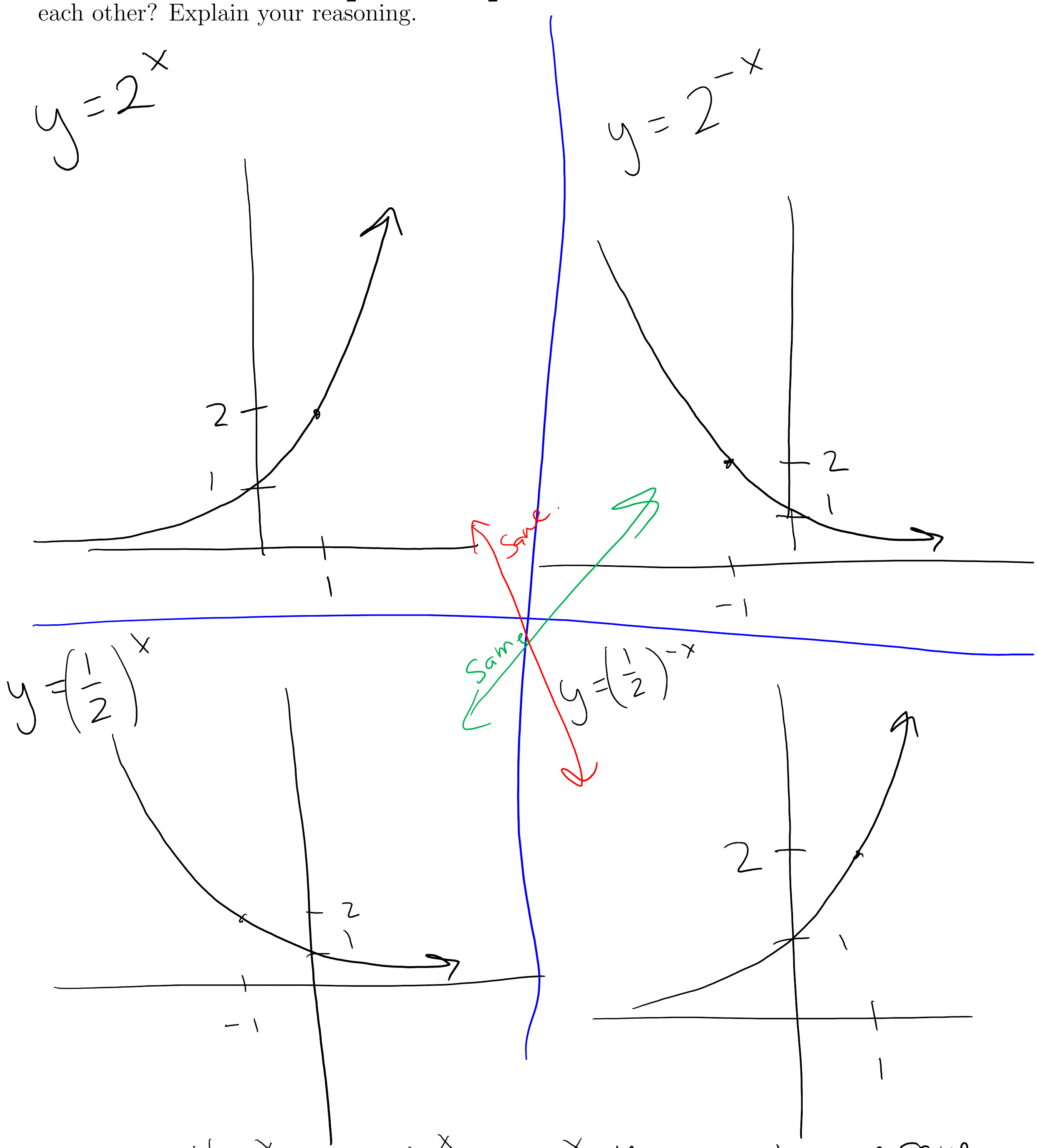
$460(1.0186)^{158} = 8459.37$ thousand

8.46 million

actual population ≈ 3.576 (census bureau 2016 estimate)

WAY off! the problem stated only growing exponentially 1860-1950. can't use for 2018.

5. Graph $y = 2^x, y = 2^{-x}, y = \frac{1}{2}^{x}$, and $y = \frac{1}{2}^{-x}$. Are any of these graphs the same as each other? Explain your reasoning.



since $\left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$ these graphs are same
similar for $\left(\frac{1}{2}\right)^{-x} = (2^{-1})^{-x} = 2^x$

More Practice from Textbook 1.3: You should do as many problems from each set (1-8, 9-16,17-24, 25-38, 39-71, 72-75, 76-77,78, 79-82), as needed until you are comfortable with these techniques. 72-75 and 79-82 are good practice for more complicated application problems.