Section 1.2: Mathematical Models
Section Objectives:

- Work through the 3 basic steps related to mathematical modeling: formulation, mathematical manipulation and evaluation.
- Know the difference between fixed and variables costs. Use these to create a linear cost model.
- Know the relationship between fixed and variable costs and the slope and y-intercepts on a linear cost model.
- Know how to find the revenue and profit functions.
- Know how to find the break-even quantity graphically and algebraically.
- Understand the demand and supply curves and know how to find the equilibrium points, quantity and price. Relate these to the ideas of surplus and shortages.
- Know how to get quadratic revenue functions from linear demand functions.
- Put quadratic functions in standard form to find the minimum or maximum.

Practice Problems

1. Jake notices there is a demand on campus for 3D printed trinkets so he decides to set-up a 3D printing shop in his dorm room. He buys a FlashForge Finder 3D from Amazon for $399. Each trinket requires $1 worth of filament to print.

   (a) Using a linear cost model, find the cost equation $C(x)$ to print x trinkets.

   $C(x) = \text{fixed} + \text{variable}$

   $\boxed{C(x) = 399 + 1 \cdot x}$

   (b) Assuming there are many other 3D print services available, we can use a linear revenue model. If Jake sells each trinket for $3, what is is revenue $R(x)$ and his profit $P(x)$.

   $R(x) = \text{price} \cdot \text{units sold} = \boxed{3x}$

   $P(x) = \text{Revenue} - \text{Cost} = 3x - (399 + x) = 2x - 399.$

   (c) How many trinkets does Jake need to sell to break even?

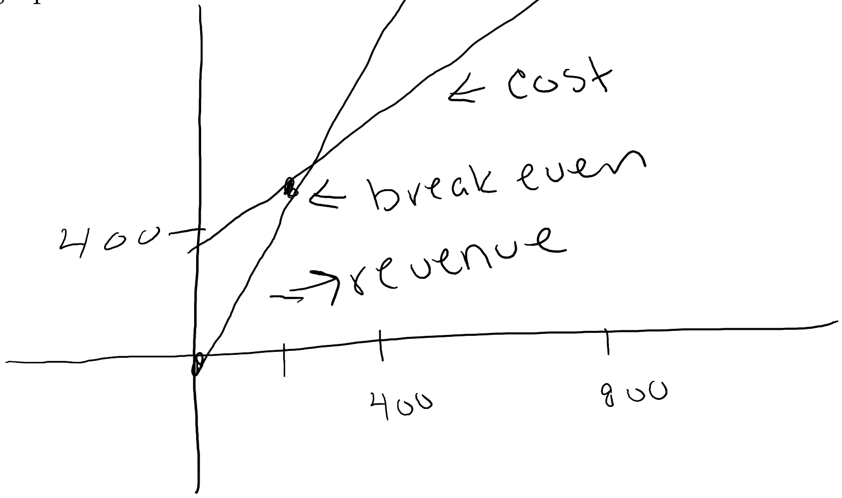
   Break even when profit $= 0$.

   Set $2x - 399 = 0$

   $2x = 399 \Rightarrow x = 199.5$ (Round to nearest integer)

   Break even when sell $\boxed{\text{200 units.}}$

(d) Graph the Revenue and Cost functions and indicate the break even point on your graph.



2. Dante is inspired by his roommate money making ventures and decides to setup his own shop. He designs and sells custom UConn eyewear which are a hit a basketball games. Since he is the sole provider, his prices are affected by supply and demand.

(a) When he prices the eyewear at \$10, he finds he can sell 120 per game. When they are \$8, he can sell 150 per game. Assuming a linear demand model, find the demand equation.

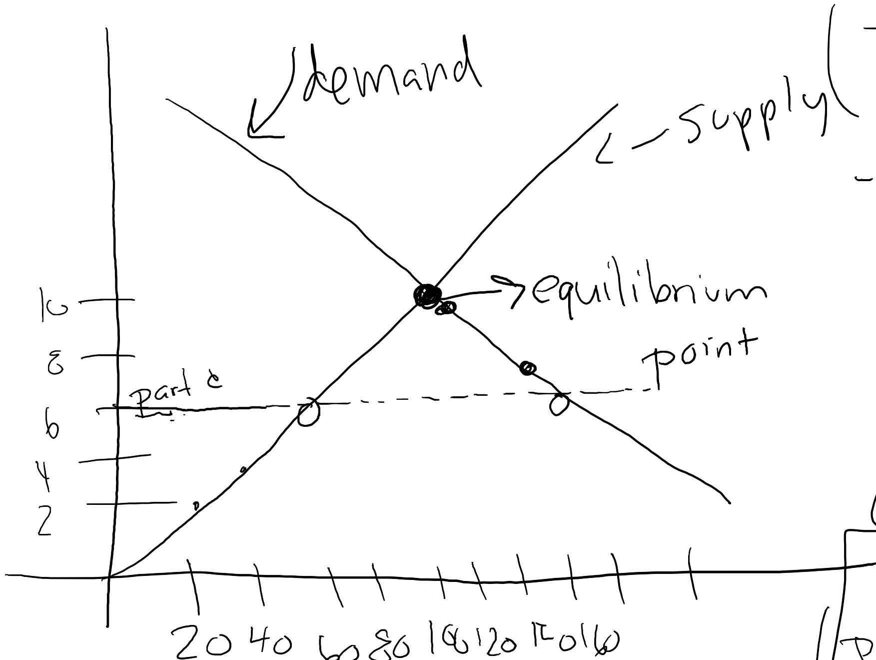
points $(120, 10)$ & $(150, 8)$

slope $\dfrac{8-10}{150-120} = \dfrac{-2}{30} = -\dfrac{1}{15}$

$p(x) = -1/15x + 10 + 8$

$p - 8 = -\dfrac{1}{15}(x - 150) \Rightarrow \boxed{p(x) = \dfrac{-1}{15}x + 18}$

(b) Dante is willing to spend more time making the eyewear if he knows he will be able to sell it for more. His supply equation is $p(x) = \dfrac{1}{10}x$. Sketch a graph of the supply and demand curves and find the equilibrium point. What is the equilibrium quantity and price?



$\left(-\dfrac{1}{15}x + 18 = \dfrac{1}{10}x\right) \cdot 30$

$-2x + 540 = 3x$

$+2x \qquad +2x$

$540 = 5x$

$x = 108$

equilibrium quantity

equilibrium price is

$p(x) = \frac{1}{10}(108) = \$10.80$

(c) UConn Athletics decides they will only allow the eyewear to be sold for \$6. At this price will there be a surplus or a shortage? Of how much?

At $p=6$ the demand is : $6 = -\frac{1}{15}x + 18$

$-18 \qquad -18$

: $-12 = -\frac{1}{15}x$

$\cdot -15 \qquad \cdot -15$ $\rightsquigarrow \boxed{x = 180}$

supply is : $p = \frac{1}{10}x$

$6 = \frac{1}{10}x$

$\boxed{x = 60}$

$\boxed{\text{shortage}}$ of $180 - 60 = \boxed{120}$

3. The demand equation for Slushees is $p = -\frac{x}{10} + 10$ where x is number of Slushees sold and p is price per Slushee.

(a) Find the revenue $R(x)$ from the sale of x Slushees.

$$R(x) = p \cdot x$$

$$= \left(-\frac{x}{10} + 10\right)(x) = -\frac{x^2}{10} + 10x$$

(b) Which quantity of Slushees sold produces the highest revenue? What is the cost per Slushee for that quantity? What is the overall revenue?

to maximize revenue

Complete square $-\frac{1}{10}(x^2 - 100x) = -\frac{1}{10}(x^2 - 100x + 2500 - 2500)$

$\frac{1}{2} \to 50$ squared

$= -\frac{1}{10}((x-50)^2 - 2500)$

$= -\frac{1}{10}(x-50)^2 + 250$

vertex $(50, 250)$

quantity: 50, cost per slushee $\frac{-50}{10} + 10 = 5$

Revenue: \$250

More Practice from Textbook 1.2: You should do as many problems from each set (1-6, 7-10, 11-14, 15-18,19-22, 23-26, 27-45, 46-60), as needed until you are comfortable with these techniques. 27-60 are good practice for application problems.

OR $x = -\frac{b}{2a} = \frac{-10}{2(-1/10)} = \frac{-10}{2} \cdot \frac{-10}{1} = \frac{100}{2} = 50$

Vertex at $x = 50$ $\quad R(50) = \frac{2500}{10} + 500 = 250$, proceed as above.