$$= \int 3x^4 + \frac{4}{5}x^{-2}\, dx$$

$$= \boxed{\frac{3}{5}x^5 - \frac{4}{5}x^{-1} + C}$$

$$= \boxed{3\ln|x| + 7e^x + C}$$

Let $\boxed{u = 3x^4 + 5}$

$du = 12x^3\, dx$

$\Rightarrow \boxed{3du = 36x^3\, dx}$

$\Rightarrow \int u^7 \cdot 3\, du$

$= \frac{3}{8}u^8 + C \Rightarrow \boxed{\frac{3}{8}(3x^4+5)^8 + C}$

Let $\boxed{u = x^3 + 1}$

$du = 3x^2\, dx$

$\Rightarrow \boxed{4du = 12x^2 dx}$

$\Rightarrow \int \frac{4}{u}\, du = 4\ln|u| + C$

$\Rightarrow \boxed{4\ln|x^3+1| + C}$

2. Consider the function $f(x) = 4e^{\ x^2}$ on the interval $[-2, 1]$. Calculate the left and right-hand Riemann Sums using three subintervals .

$\hookrightarrow [-2,1] \Rightarrow [-2,-1], [-1,0], [0,1]$

$L = \frac{1-(-2)}{3} = 1$

LH: $1\left(f(-2) + f(-1) + f(0)\right) = 4e^{-(-2)^2} + 4e^{-(-1)^2} + 4$

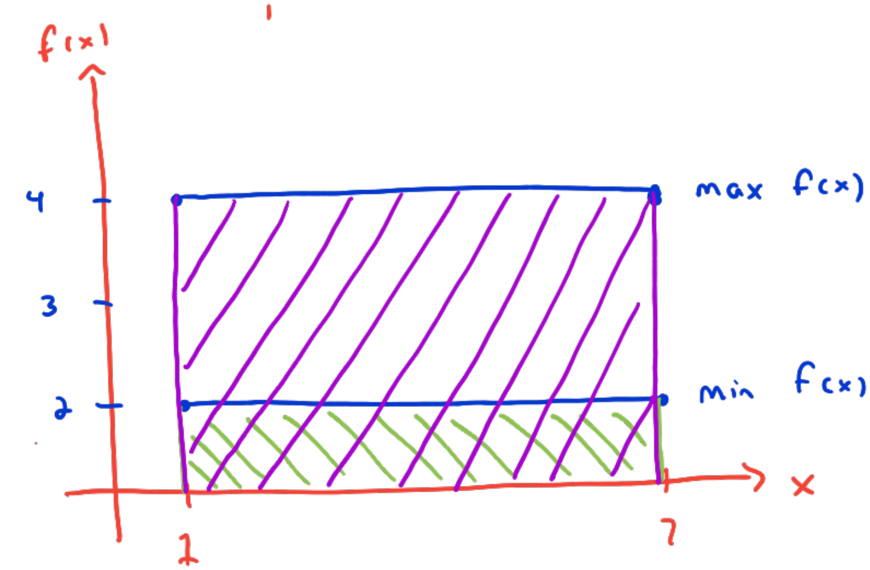
≈ 5.545

RH: $1\left(f(-1) + f(0) + f(1)\right) = 4e^{-(-1)^2} + 4 + 4e^{-(1)^2}$

≈ 6.94

3. Consider an unknown function $f(x)$ on the interval $[1, 7]$. Suppose we do know that $2 \leq f(x) \leq 4$ on $[1, 7]$. Draw a geometric representation of this information and determine the range of value $\int_1^7 f(x); dx$ could obtain.

$\int_1^7 f(x)\,dx \rightarrow$ represnts area under $f(x)$ on $[1,7]$



$\rightsquigarrow$ If $f(x) = 4$ on $[1,7]$ then

$\int_1^7 f(x)\,dx = 6(4) = 24$

$\rightsquigarrow$ If $f(x) = 2$ on $[1,7]$ then

$\int_1^7 f(x)\,dx = 6(2) = 12$

So,

$12 \leq \int_1^7 f(x)\,dx \leq 24$

4. Suppose $\int^{3}{}_{3} g(x)\,dx = -10$. We also know $\int_1^3 g(x)\,dx = 4$, what is the value of $\int^{1}{}_{3} g(x)\,dx$?

$$-10 = \int_{-3}^{3} g(x)\,dx = \int_{-3}^{1} g(x)\,dx + \int_{1}^{3} g(x)\,dx = y + 4$$

$$-10 = y + 4 \Rightarrow \int_{-3}^{1} g(x)\,dx = -14$$

5. Calculate the following definite integrals.

(a) $\int^{1}{}_{1} \sqrt[3]{x} + 4x^5 - 5\,dx$.

$$\int_{-1}^{1} x^{1/3} + 4x^5 - 5\,dx = \frac{3}{4}x^{4/3} + \frac{4}{6}x^6 - 5x \Big|_{-1}^{1} = 10$$

(b) $\int_0^{10} e^{4x}\,dx$. Can use "u"-sub w/ u = 4x or just think...

$$\Rightarrow \frac{1}{4}e^{4x}\Big|_0^{10} = \frac{1}{4}e^{40} - \frac{1}{4}$$

(c) $\int_3^5 \frac{5x}{x^2}\,dx$. Let $u = x^2 \Rightarrow du = 2x\,dx \Rightarrow \frac{5}{2}du = 5x\,dx$

$$\Rightarrow \int_{u=3^2}^{u=5^2} \frac{5}{2}\cdot\frac{1}{u}\,du = \frac{5}{2}\ln|u|\Big|_{u=9}^{u=25} = \frac{5}{2}(\ln 25 - \ln 9)$$

↕ same!

or $$\Rightarrow \int \frac{5}{2}\frac{1}{u}\,du = \frac{5}{2}\ln|u| \Rightarrow \frac{5}{2}\ln x^2 \Big|_3^5 = \frac{5}{2}(\ln 25 - \ln 9)$$

6. Suppose you own a new company that specifically targets "millennials". You decide the best course of action is to produce avocados and "truly white" seltzers. Let x be the amount of time in months since start up, the revenue function is $R(x) = x^2 - 3$ in hundreds of thousands of dollars, and the cost function is $C(x) = 2x + 5$ in hundreds of thousands of dollars. Calculate the area between the two curves on the interval $[0, 9]$ and interpret your answer in context of the question.

Find if $R(x) = C(x)$ on $[0,9]$.

$$x^2 - 3 = 2x + 5 \Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow (x-4)(x+2) = 0$$

$$\boxed{x = 4}$$

Now on $[0,4)$ $C(x) > R(x)$ and on $(4,9]$ $R(x) > C(x)$

So, Area between is

$$\int_0^4 C(x) - R(x)\,dx + \int_4^9 R(x) - C(x)\,dx$$

$$= \int_0^4 -x^2 + 2x + 8\,dx + \int_4^9 x^2 - 2x - 8\,dx$$

$$= 26\tfrac{2}{3} + 116\tfrac{2}{3}$$

Be careful, both value represent net profit, except on $[0,4)$ $C(x) > R(x)$ so $26\,^2/_3$ counts as a loss.

Total profit is \$90 thousand in the first 9 months.