Complete the following problems in preparation for your second exam.

1. Suppose that the when the JBL Flip 5 is priced at \$100 it has an elasticity of 1.75. That is, $E(100) = 1.75$. If the price of the speaker is raised to \$120, what is the approximate change in demand for the Flip 5? Does this change represent an increase or decrease in demand?

$$E(p) = \frac{\%\ \Delta \text{ in demand}}{\%\ \Delta \text{ in price}} \Rightarrow E(100) = 1.75 = \frac{\%\ \Delta \text{ in } D}{\%\ \Delta \text{ in } P}$$

$$\%\ \Delta \text{ in price} = \frac{120 - 100}{100} = \frac{20}{100} \Rightarrow 20\% \text{ change}$$

$$E(100) = 1.75 = \frac{\%\ \Delta \text{ in } D}{20\%} \Rightarrow \%\ \Delta \text{ in } D = (1.75)(20\%)$$

$$= \boxed{35\% \text{ decrease in demand}}$$

2. The function below represents the wealth of a hedge-fund manager in millions where t represents time in years since the manager started working on Wall Street. Find the open intervals where this function is increasing and decreasing. What does this represent in context of the question?

$$W(t) = 0.25 \ln\left(\sqrt{(t + .75)^5}\right).$$

$$\frac{d}{dx}\left(\ln(f(x))\right) = \frac{f'(x)}{f(x)}$$

$$0.25 \ln\Big(\underbrace{(t + 0.75)}_{f(t)}{}^{5/2}\Big)$$

$$\Rightarrow f'(t) = \frac{5}{2}(t + 0.75)^{3/2}(1)$$

Want

$$W'(t) = 0.25 \frac{\frac{5}{2}(t + 0.75)^{3/2}}{(t + 0.75)^{5/2}}$$

$$= \frac{5}{8} \cdot \frac{1}{(t + 0.75)}$$

Also, $t \geq 0$

For any $t > 0$, $W'(t) = \frac{5}{8(t + 0.75)} > 0$

C.U. $t = -0.75$

* Is not in the domain! No C.U.

$$\Rightarrow \boxed{W(t) \text{ is increasing on } (0, \infty).}$$

→ Represents long-term behavior

3. Find all vertical and horizontal asymptotes of the following function,

local behavior

$$f(x) = \frac{3x^2 - 12x}{x^2 - 2x - 3}.$$

→ $3x(x-4)$

V.A.

When the denominator is 0!

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\underbrace{x = 3, -1}$

Both V.A.

HA

$$\lim_{x \to \pm\infty} \frac{3x^2 - 12x}{x^2 - 2x - 3} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$\lim_{x \to \pm\infty} \frac{3 - \cancel{\frac{12}{x}} \to 0}{1 - \cancel{\frac{2}{x}} \to 0 - \cancel{\frac{3}{x^2}} \to 0} = \frac{3}{1}$$

$y = 3$ is a H.A.

4. The following function represents the marginal ~~cost~~ Profit function for "soda" sold at the Rent on a given game day. Calculate $P''(x)$ and find intervals of increasing and decreasing of $P'(x)$. What does this tell you about $P(x)$?

$$P'(x) = x^2 + 20x + 2.$$

$P''(x) = 2x + 20 \Rightarrow x = -10$ is a C.V. of $P'(x)$.

If $x \geq 0 \Rightarrow P''(x) > 0$, $P'(x)$ is increasing on $(0, \infty)$.

Marginal Profit function for soda is increase, i.e. the rate at which Profit changes is growing!

$P(x)$ on $(0, \infty)$ is a concave up function!

* 1^{st} derivative ⇒ rate of change
* 2^{nd} derivative ⇒ concavity

|| P'' is the 1^{st} derivative of P'.

5. On the given plot, graph a function $f(x)$, that satisfies the following properties (be sure to include labels to make your picture clear):

- $\boxed{\lim_{x\to-\infty} f(x) = -\infty}$ and $\lim_{x\to\infty} f(x) = 3$ → long term behavior
- $\lim_{x\to1^-} f(x) = \infty$ and $\lim_{x\to1^+} f(x) = -\infty$ → Vertical asymptote
- $f'(x) > 0$ on $(-\infty,-3)\cup(0,1)\cup(1,2)$ and $f'(x) < 0$ on $(-3,0)\cup(2,\infty)$. inc/dec
- $f''(x) > 0$ on $(0,1)\cup(3,\infty)$ and $f''(x) < 0$ on $(-\infty,0)\cup(1,3)$. Concavity
- $f(x)$ has a relative max at $(-3,2)$ and $(2,5)$ and a relative minimum at $(0,-2)$.

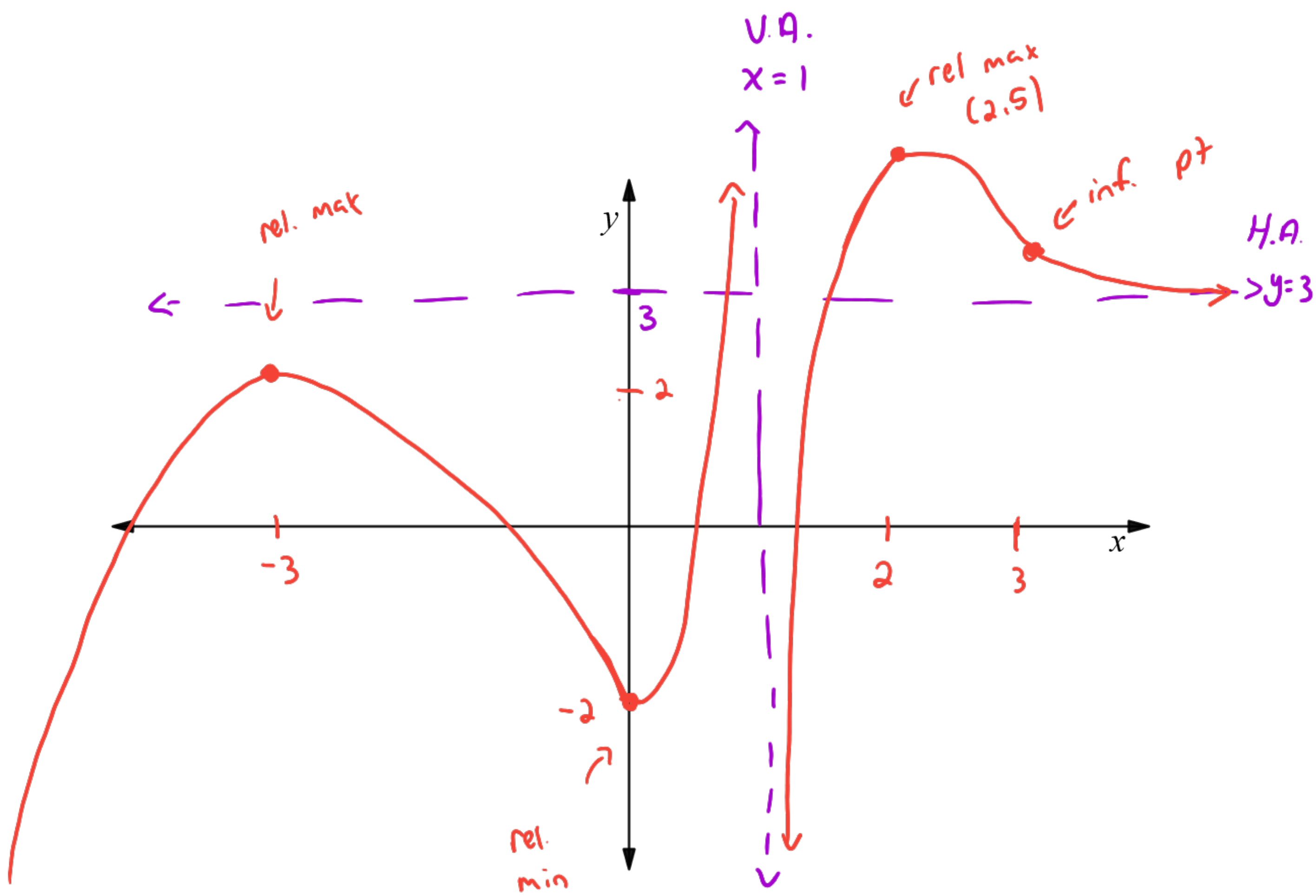
$(-\infty,-3)$ → inc/CCD

$(-3,0)$ → dec/ccd

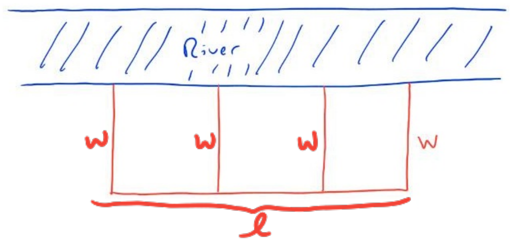
$(0,1)$ → inc/ccu

$(1,2)$ → inc/CCD

$(2,\infty)$ → dec w/ change from CCD to CCU at $x=3$



6. A rectangular field of area $10,000\ ft^2$ next to a river is to be subdivided, into three congruent pens, as shown in the diagram. Assume the river makes the fourth side. Assume the fencing costs \$15 per foot. Find the dimensions of the pens that will minimize the cost.



Enclose 10,000 ft^2 with the smallest perimeter!

$A = 10,000 = lw \Rightarrow$ constraint eqn

$P(l,w) = 4w + l \Rightarrow$ Function to minimize

$\frac{10,000}{w} = l \Rightarrow P(w) = 4w + 10000w^{-1}$

$w > 0,\ w < \infty,$ Minimize $P(w)$ on $(0,\infty)$

$$P'(w) = 4 - 10000w^{-2} = 0$$

$$= \frac{-10000}{w^2} = -4$$

$$\Rightarrow 10000 = 4w^2$$

$$\Rightarrow w = \pm\sqrt{2500}$$

$$w = \pm 50 \Rightarrow w = 50\,ft$$

$(0,50)$	$(50,\infty)$
$P'(1) < 0$	$P'(51) > 0$

$\boxed{w = 50\ ft \text{ minimizes the problem!}}$