

Complete the following problems in preparation for your first exam.

1. Suppose you would like to save \$25,000. If you deposit \$20,000 into an account paying 2.24% interest compounded continuously. How much time does the money need to stay in the account in order to reach your goal. Use logarithms to solve.

Need:  $A = Pe^{rt}$  and solve for  $t$ .

$$25,000 = 20,000 e^{0.0224 t}$$

$$1.25 = e^{0.0224 t}$$

$$\ln(1.25) = \ln(e^{0.0224 t})$$

$$\ln(1.25) = 0.0224 t$$

$$\Rightarrow t = \frac{\ln(1.25)}{0.0224} \approx 10 \text{ years.}$$

2. Use properties of logarithms to expand the following expression into the sum, difference, or constant multiples of logarithms.

$$\log_6 \left( \frac{\sqrt{x}(y-1)}{x^5 y} \right)$$

$$= \log_6 (\sqrt{x}(y-1)) - \log_6 (x^5 y)$$

$$= \log_6 (x^{1/2}) + \log_6 (y-1) - \log_6 (x^5) - \log_6 (y)$$

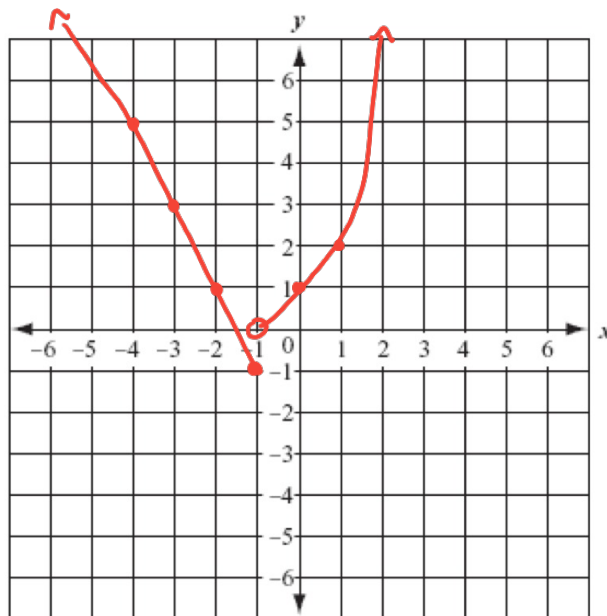
$$= \frac{1}{2} \log_6 (x) + \log_6 (y-1) - 5 \log_6 (x) - \log_6 (y)$$

3. Let  $f$  be given by:

$$f(x) = \begin{cases} -2x - 3 & \text{if } x \leq -1 \\ x^3 + 1 & \text{if } x > -1 \end{cases}$$

(a) Evaluate  $f(-1)$ .

(b) Sketch a graph of the function  $f$ . **Label the  $(x, y)$ -coordinates of at least one point on each piece on the graph.**



(c) Write the interval(s) for which  $f$  is decreasing. **Use interval notation.**

$$(-\infty, -1)$$

(d) Is  $f(x)$  a continuous function? Explain.

No, at  $x = -1$ , there is a jump in the graph.

Also,  $\lim_{x \rightarrow -1} f(x)$  DNE

4. The following function represents the profit function for "soda" sold at the Rent on a given game day.

\* typo  $\rightarrow C(x) = x^2 - 200x + 2$ . Replace w/  $P(x)$ ...

- (a) Using the limit definition of the derivative, calculate  $C'(x)$ .

$$\lim_{h \rightarrow 0} \frac{C(x+h) - C(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 200(x+h) + 2 - (x^2 - 200x + 2)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\underbrace{x^2 + 2xh + h^2}_{\text{green}} - \underbrace{200x - 200h}_{\text{blue}} + \underbrace{2}_{\text{purple}} - \underbrace{x^2}_{\text{green}} + \underbrace{200x}_{\text{blue}} - \underbrace{2}_{\text{purple}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 200h}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h - 200 \Rightarrow 2x - 200$$

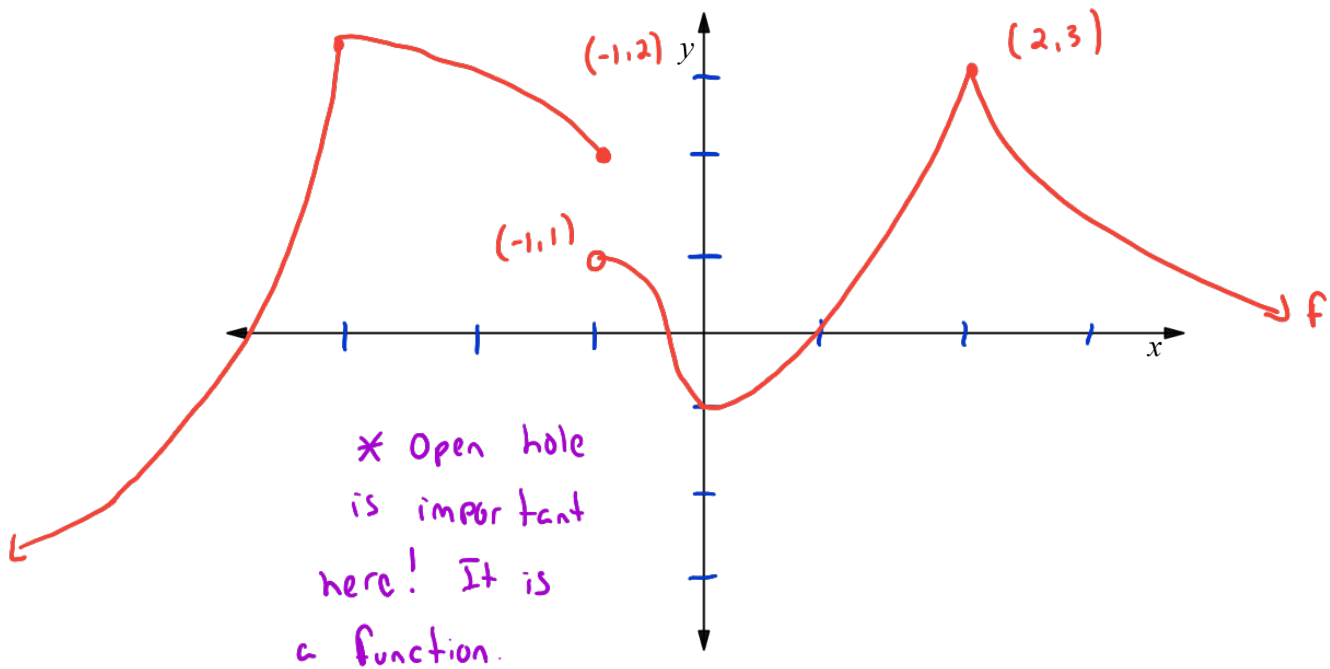
$$\Rightarrow C'(x) = 2x - 200$$

- (b) Evaluate  $P'(101)$  and explain what it means in context of the question.

$P'(101) = 2$ . This means, once 101 "sodas" are sold, profit is increasing at a rate of \$2 per "soda"

5. On the given plot, graph a function  $f(x)$ , that satisfies the following properties (be sure to include labels to make your picture clear):

- $f(x)$  is increasing on  $(-\infty, -3) \cup (0, 2)$  and decreasing on  $(-3, 0) \cup (2, \infty)$ .
- $\lim_{x \rightarrow 2} f(x) = 3$ ,  $f(x)$  is continuous at  $x = 2$ , but  $f'(x)$  is **undefined** at  $x = 2$ .
- $\lim_{x \rightarrow (-1)^-} f(x) = 2$ .
- $\lim_{x \rightarrow (-1)^+} f(x) = 1$ .



6. Suppose a Tax Advisor makes \$40 per hour. In addition, if she works more than 40 hours in one week she will be paid time and a half. That is, any time over 40 hours the advisor will be paid \$60 per hour. Write a piecewise function that describes her pay if she works  $t$  number of hours.

$$P(t) = \begin{cases} 40t, & 0 \leq t \leq 40 \\ 60(t-40) + 1600, & t > 40 \end{cases}$$

\$60 for each hour past 40.  
 fixed pay, first 40 hrs.