1 Exam 1 Material

Sections A.1, A.2 and A.6 were review material. There will not be specific questions focused on this material but you should know how to: Simplify functions with exponents. Factor quadratics to solve for zeros. Rationalize the numerator/denominator. Find the equation for linear functions given a point and a slope or two points. All of these skills are tested throughout the course.

Section 1.1 Functions

You should know how to find the domain of a function and evaluate functions at specific values or variables.

1. What is the domain of the function below?

$$\frac{x+\sqrt{1-x}}{x^2+5x+4}$$

2. If $f(x) = 3x^2 + 2x$, what is $f(3+h)$?

More practice: 11-24, 32-38, 51-58

Section 1.2: Mathematical Models

Find equations for cost, revenue and profit (using linear models). Understand the basics of the supply and demand curves. Find equilibrium and break-even points.

3. A cost model is given by $3x + 400$. What were the fixed costs? What was the cost to make each additional item? If you sell each item for \$5, what are your revenue and profit functions? What is your break even point?

4. The revenue function for a certain product is $R(x) = -2x^2 + 15x$. The cost function is $C(x) = 5x + 8$. What is the break-even quantity?

More practice: 1-22, 29

Section 1.3: Exponential Models

Know the formula for continuously compounded interest $A = Pe^{rt}$ and how to apply it. (You do not need to know the formula for interest compounded n times for year.) Know how to find the present value.

5. A bank account earns 4% interest compounded continuously. How much money should you put in so that you have \$5,000 after 5 years?

More practice: 39-49 (only compounded continuously)

Section 1.4: Composition of Functions

Know how to identify a function as the composition of two other functions (this was important later when we talked about the chain rule). Know how to find the domains of function obtained under composition, addition, subtraction, multiplication and division.

6. Let $f(x) = 3x^2$ and $g(x) = 2x + 1$. Find f/g, g/f, $f \circ g$ and $g \circ f$ and each of their domains.

7. What functions are used to make the following composition $\ln(2x^2 + 1)$? (i.e. what is the "outside function" and what is the "inside function"?

More practice: 1-30

Section 1.5: Logarithms

Know the definition of a logarithm as the inverse of an exponential. Know how to solve equations using involving logarithms by using exponentials. Know the basic logarithm rules.

8. If we start with \$5000 in a bank account with interest compounded continuously and want \$8000 in ten years, what interest rate do we need?

9. Solve for $x : 2\ln(x+1)+1=2$.

More practice: 1-40, 48-50

Section 3.1: Limits

Know how to solve limits algebraically. This include techniques of plugging in numbers, factoring the numerator and/or denominator and cancelling like terms and rationalizing the numerator or denominator. (We later learned different techniques for limits as $x \to \infty$.) We also talked about algebraic rules for limits and finding one and two sided limits from graphs and tables. We defined continuity of functions using limits. Know the limit definition of a vertical asymptote (i.e either of the one sided limits or the two sides limit as $x \to a$ is either ∞ or $-\infty$

10. Evaluate
$$\lim_{x \to 5} \frac{x^2 + 3x + 2}{x + 2}.$$
Is there a vertical asymptote at $x = 5$, why or why not?

11. Evaluate
$$\lim_{x \to -2} \frac{x^2 + 3x + 2}{x + 2}.$$
Is there a vertical asymptote at $x = -2$, why or why not?

12. Evaluate
$$\lim_{x \to -1} \frac{x^2 + 3x + 2}{x + 1}.$$
Is there a vertical asymptote at $x = -1$, why or why not?

13. Evaluate
$$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}.$$

14. If $\lim_{x\to 3} f(x) = 7$ what is $\lim_{x\to 3} \sqrt{4f(x)+2}$?

15. Sketch the graph of a function with $f(1) = 2$, $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 4$

More Practice: 1-46

Section 3.2: Rates of Change

Know the formula for average rate of change on an interval and for instantaneous rate of change which gives the slope of the tangent line which we later called the derivative

16. Let $f(x) = 3x^2 + 2$. Find the average rate of change on the interval [1,4].

17. Let $f(x) = 3x^2 + 2$. Find the instantaneous rate of change at $x = 2$. (Check your work using derivative rules) Then find the equation of the line tangent to the curve at $x = 2$.

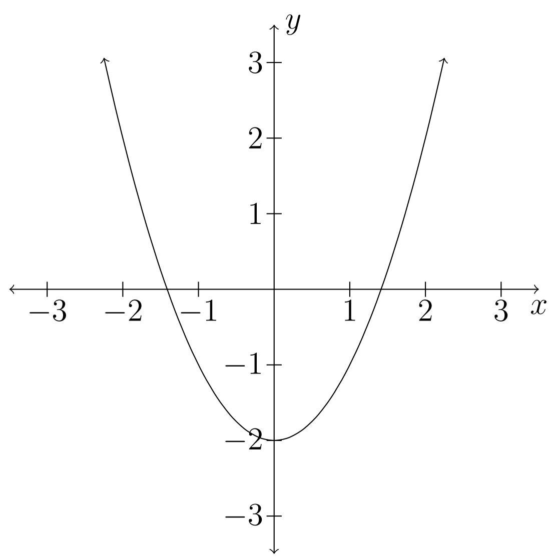
More practice: 1-32

Section 3.3: The Derivative

Know the limit definition for the derivative. Know how to use the derivative to find equation of tangent line (like above). Know when a function is not differentiable (corner, cusp, vertical tangent or discontinuity). Interpret the graph of the derivative to tell us things about the function (and the graph of the function to tell us about the derivative).

18. Let $f(x) = x^3$. Find $f'(x)$ using the limit definition of derivative.

19. A graph of a function is below. Use this to sketch the graph of its derivative.



2 Exam 2 Material

4.1-4.4: Derivative Rules

Know how to find the derivative of functions using the power rule, e^x rule, sum rule, product and quotient rules, the chain rule and rules for logarithms. Know that the derivative of cost gives marginal cost (approximate cost for next item) and of revenue gives marginal revenue (revenue generated by next item).

20. Find the derivative of the following functions:
    (a) $f(x) = 3x^3 + 1/x + 5e^x$

    (b) $f(x) = (3x^2 + 2x)(\ln(e^x + 1))$

    (c) $f(x) = \dfrac{3x^2e^x}{\ln(3x - 2)}$

21. The cost to produce a product is given by $C(x) = \frac{4}{x-1}$. Find and interpret $C'(3)$.

More practice: Chapter 4 Review Exercises (page 290) 1-38

3.4: Local linearity

Find the equation of the tangent line to a curve. Use the tangent line to approximate the function. Think of this as using marginal cost/revenue to to help approximate cost/revenue.

22. Let $C(x) = 1/2x^2 + 2x + 4$ be the cost to produce x pounds of a product. Find and interpret $C'(4)$. Then find the equation of the tangent line at $x = 4$. Use this to approximate the cost to produce 5 items. Illustrate your answers with a graph.

More practice: 1-30

4.5: Elasticity of Demand

Know the formula for elasticity of demand (E) and its interpretation as $\frac{\%\text{decrease in demand}}{\%\text{increase in price}}$. Use this interpretation to solve problems. Know when demand is elastic, inelastic or unit elastic. Know that revenue is optimized when $E = 1$.

23. Let $x = \frac{1}{10+p}$. Find the elasticity of demand when $p = 5$.

24. For a certain product $E = 3$. When the price if \$4, 300 items are sold. If price decreases from \$4 to \$3, how many items do we expect to sell?

25. If the demand for a product is given by $30 - 10p$, what price maximizes revenue?

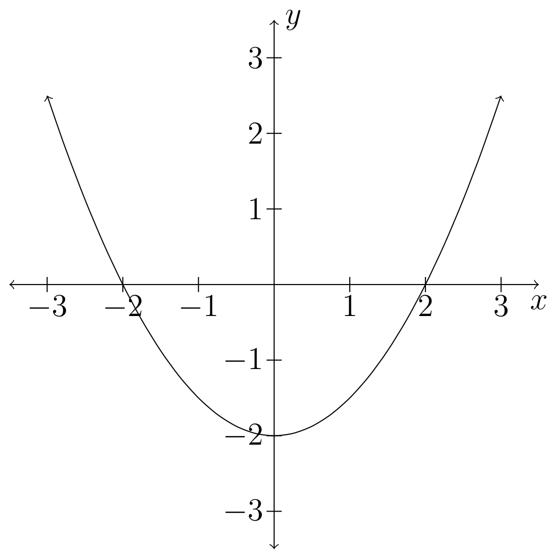
Section 5.1: The First Derivative

Know how to use the first derivative of a function to find the intervals where it is increasing and decreasing. Identify any relative minima or maxima.

26. Let $f(x) = 3x^5 - 5x^3 + 1$. Find the interval where f is increasing and where f is decreasing. Identify any relative minima or maxima.

27. Let $g'(x) = \dfrac{3+x}{x-2}$. Find the interval where g is increasing and decreasing. (Note: you have been given the derivative already). Identify any relative minima or maxima.

28. The graph of $h'(x)$ (the graph of the derivative) is given below. Where is h increasing and where is h decreasing? Identify any relative minima or maxima.



More Practice: 1-44

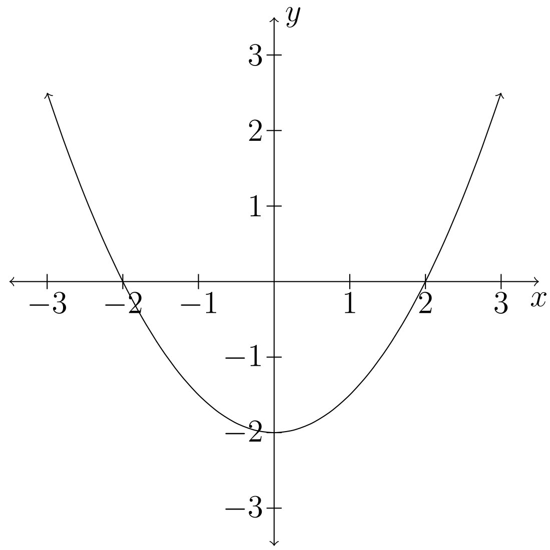
Section 5.2: The Second Derivative

Know how to use the second derivative to determine if a function is concave up/concave down and the location of inflection points.

29. Let $f(x) = 3x^5 - 5x^3 + 1$. Find the intervals where f is concave up and where f is concave down. Identify any inflection points.

30. Let $g'(x) = \dfrac{3+x}{x-2}$. Find the intervals where g is concave up and where g is concave down. Identify any inflection points. (Note: you have been given the first derivative already).

31. The graph of $g'(x)$ (the graph of the derivative) is given below. Find the intervals where g is concave up and where g is concave down. Identify any inflection points.(Note: you have been given the first derivative already).



More practice: 1-6, 13-41

Section 5.3 Limits at Infinity

Know the algebraic technique (divide by highest power of x on numerator and denominator) to find the limit of a rational function as $x \to \infty$ or $x \to -\infty$. Know how to find the limits as $x \to \pm\infty$ for polynomials and exponentials. Know the limit definition of a horizontal asymptote. Remember: a graph can pass through a horizontal asymptote, it just must approach it as either x goes to ∞ or as x goes to $-\infty$ or both.

32. Evaluate the following limits using algebraic techniques. Identify any horizontal asymptotes.

    (a) $\lim_{x\to\infty} \dfrac{3+3x}{x-2}$

    (b) $\lim_{x\to\infty} \dfrac{x^2+2x-1}{x-4}$

    (c) $\lim_{x\to\infty} \dfrac{x^3+2x+1}{x^3-2x}$

    (d) $\lim_{x\to\infty} -2x^3$

More practice: 1-18

Section 5.4: Curve Sketching

Sketch a graph of a curve using the appropriate: domain, symmetries, asymptotes, intercepts, increasing/decreasing behavior, relative min/max, concave down/concave up behavior and inflection points.

33. Sketch a graph of $f(x) = \frac{x^2-1}{3x^2}$ using all of the information above.

34. While analyzing a function you determine the following facts. Use these to sketch a graph of the function.

- The only intercepts of the function are an x-intercept of 4 and a y-intercept of -2
- There are vertical asymptotes at $x = -3$ and $x = 3$.
- $\lim_{x\to-\infty} f(x) = -1$ and $\lim_{x\to\infty} f(x) = -1$.
- The function is decreasing on $(-\infty, -3) \cup (2, 3) \cup (3, \infty)$. It is increasing on $(-3, 2)$. There is a relative max at $(2, -1/2)$.
- The second derivative is positive on $(0, 1) \cup (3, \infty)$ and negative on $(-\infty, -3) \cup (-3, 0) \cup (1, 3)$.

More Practice: 1-36

Section 5.5: Absolute Extrema

Know that for a continuous function the absolute extrema can only occur at critical points or points on the boundary of the interval. If the boundary points are not included in the interval or the function is not continuous, then the absolute extrema might not exist.

35. Find the absolute minimum and maximum value of the following function on the indicated interval, if it exists.
$$f(x) = x^2 - 4x \text{ on } (-2, 5].$$

36. Find the absolute minimum and maximum value of the following function on the indicated interval, if it exists.
$$f(x) = \frac{1}{3x} \text{ on } [-2, 4].$$

37. Find the absolute minimum and maximum value of the following function on the indicated interval, if it exists.
$$f(x) = \frac{1}{3x} \text{ on } [2, 4].$$

More Practice: 1-28

Section 5.6: Optimization and Modeling

Set-up and solve optimization problem by reducing the function you want to minimize or maximize to one variable and then taking the derivative to find the min or max.

38. A company is creating frames to hand out at an event. The perimeter of the frame must be 44 inches. The minimum length of each side is four inches. What is the minimum and maximum area of the frame?

39. A fence is built around a 200sqft garden. The cost for one of the sides is \$10 per foot. The cost of the other 3 sides is \$3 per foot. Find the dimensions that minimize cost.

More Practice:1-18

3 Material Since Exam 2

Section 6.1: Antiderivatives

Know how to find the general antiderivatives of functions using the power, constant multiple, sum, and exponential rules, and special rule for $1/x$. Know that the antiderivative of marginal revenue gives the revenue function and of marginal cost gives cost.

40. Find the general antiderivative of the following functions.

    (a) $\displaystyle\int e^{2x} + x^2 - \frac{2}{x}\,dx$

    (b) $\displaystyle\int 4x^3 - \frac{2}{x^2} + 4\sqrt{x}\,dx$

41. The marginal revenue for a certain product is given by $R'(x) = 3x^2 + 2\sqrt{x}$. Find the revenue function.

42. The marginal cost for a certain product is given by $C'(x) = x + e^x$. Find the cost function if the fixed costs are \$100.

Section 6.2: Substitution

Know how to use the method of substitution to evaluate indefinite (and definite) integrals. (Definite integrals were covered later, but you should know how to evaluate them using substitution).

43. Evaluate the following definite and indefinite integrals:

(a) $\displaystyle\int \frac{x^2}{\sqrt{x^3+4}}\,dx$

(b) $\displaystyle\int e^{3x^2+2x}(3x+1)\,dx$

(c) $\displaystyle\int_1^3 \frac{[\ln(x)]^4}{x}\,dx$

Section 6.4: The Definite Integral

Know how to estimate the area under the curve by finding the area of (a small number of) rectangles under the curve using either left-hand endpoint, right-hand endpoints or midpoints. Know how to find the exact area geometrically.

44. Let $f(x) = x^2 + 1$. Estimate $\int_0^6 f(x)\,dx$, using 3 rectangles and the indicated endpoints. Give a sketch of the estimated area in each case.

(a) right endpoints

(b) left endpoints

(c) midpoints.

45. Evaluate $\int_4^6 4x - 2\,dx$ exactly, using geometry.

Section 6.5: The Fundamental Theorem of Calculus

Know the geometric properties of definite integrals. Know how to use both part 1 and part 2 of the Fundamental Theorem of Calculus.

46. If $\int_1^3 f(x)\,dx = 4$ and $\int_3^6 f(x)\,dx = 3$ and $\int_1^6 g(x)\,dx = 1$, what is $\int_1^6 2f(x) + 3g(x)\,dx$?

47. Evaluate $\int_4^6 4x - 2\,dx$ using the Fundamental Theorem of Calculus, part 2.

48. Evaluate $\displaystyle\int_1^5 \frac{x^2}{4x^3+2}\,dx$.

49. Find $\displaystyle\frac{d}{dx}\int_4^x \frac{e^{3t}}{t^2+1}\,dt$.

Section 6.6: Area between two curves

Know how to find the area between two curves but integrating the top curve minus the bottom curve. Know how to find the limits of integration by finding the intersection points between the two curves.

50. Sketch and find the area of the region bounded by the curves $f(x) = x^2 - 5$ and $g(x) = x + 1$.