

This is an example of an exam that was given in a previous semester. This exam is being provided as a study aid.

- You should NOT expect the actual exam to be the same as this exam.
- Some topics may be emphasized more or less. Some topics that are not covered on this exam might be covered on yours.
- Knowing just how to do the questions on this exam is not a good study technique.
- It is recommended that you complete this as a practice exam under as close to exam conditions as possible - give yourself 50 minutes in a quiet room after studying. Do not look at notes while taking the exam. Once you are done, go back and review topics you had trouble with. Then, use the worksheets for more practice or to identify other topics to review.

Happy Studying!

1. Evaluate the derivative of each of the following functions. You do not need to simplify your answers. [12]

(a) $3x^2 + \ln(x) + 5e^x$

$$6x + \frac{1}{x} + 5e^x$$

(b) $(x^2 + 3x + 2) \ln(3x^2)$

$$(x^2 + 3x + 2) \frac{6x}{3x^2} + (2x + 3) \ln(3x^2)$$

(c) $\frac{(x^2 + 3x)^4}{e^{4x}}$

$$\frac{e^{4x} \left(4(x^2 + 3x)^3 (2x + 3) \right) - (x^2 + 3x)^4 \cdot 4e^{4x}}{(e^{4x})^2}$$

2. The elasticity of demand for a product is 4. We know if the price is \$100, we can expect to sell 200 items. If we increase the price from \$100 to \$103, how many do we expect to sell? Explain your reasoning.

[5]

$P = 100$ demand = 200
 ↓
 103
 ↑
 3% increase in price • 4 = 12% decrease in demand
 elasticity in demand

new demand: $200 - 12\%(200) = 200 - 24 = \boxed{176}$

3. The second derivative of a function defined on $(-\infty, \infty)$ is given by

[6]

$$f''(x) = \frac{(x-1)(x+3)}{(x-2)}$$

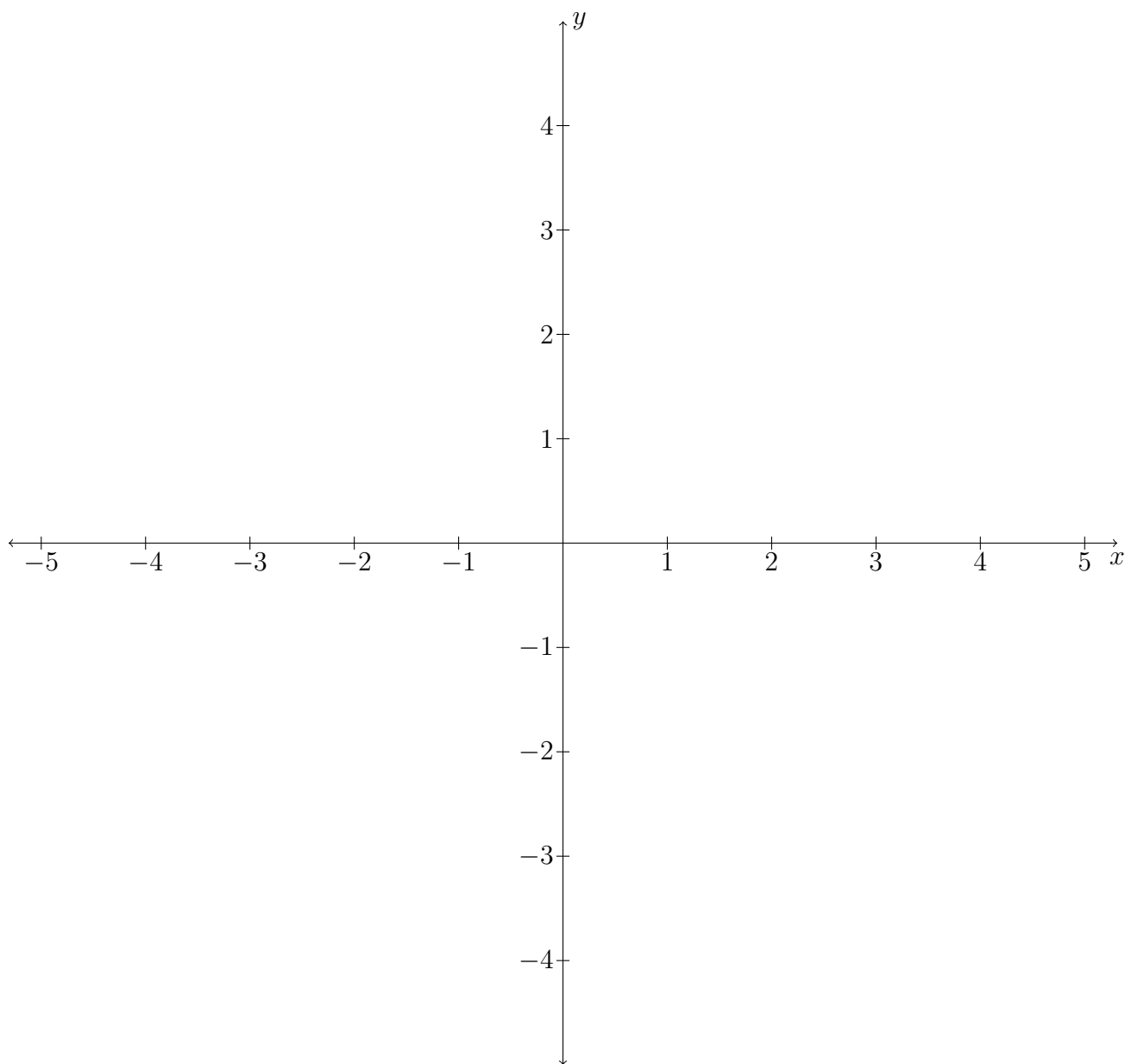
Find the intervals where f is concave up and those where it is concave down.

$$f''(x) = 0 \quad \text{when } x = 1, -3$$

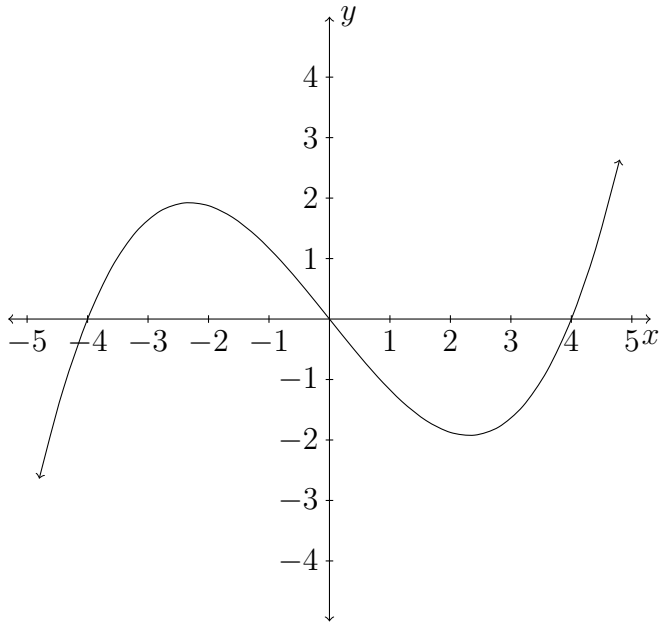
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4. While analyzing a function you determine the following facts. Use these to sketch a graph of the function on the axes provided. [10]

- The only intercepts of the function are x -intercepts of 0 and -4 and a y -intercept of 0.
- There are vertical asymptotes at $x = -3$ and $x = 3$.
- $\lim_{x \rightarrow -\infty} f(x) = 2$ and $\lim_{x \rightarrow \infty} f(x) = 2$.
- The function is decreasing on $(-\infty, -3) \cup (0, 3) \cup (3, \infty)$. It is increasing on $(-3, 0)$. There is a relative max at $(0, 0)$.
- The second derivative is positive on $(3, \infty)$ and negative on $(-\infty, -3) \cup (-3, 3)$.



5. The graph of the **first derivative of $f(x)$** is given below. Use this to determine when $f(x)$ is increasing and when $f(x)$ is decreasing. [5]



6. Let $f(x) = 2e^{x^2-x}$. Find the absolute minimum on $[0, 1]$. Explain your reasoning with calculus. [5]

7. Let

$$f(x) = \frac{x^2 - 2x - 3}{x - 2} = \frac{(x - 3)(x + 1)}{(x - 2)}$$

- (a) Find the equations of the vertical asymptotes of $f(x)$. Use limits to explain your answer. Use algebraic techniques to evaluate the limit, where appropriate. [4]

- (b) Find the equations of the horizontal asymptotes of $f(x)$. Use limits to explain your answer. Use algebraic techniques to evaluate the limit, where appropriate. [4]

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8. An apartment complex can rent all 150 of its one-bedroom apartments at a monthly rate of \$650. For each \$10 increase in rent, 2 additional apartments are left unoccupied.
- (a) How many apartments will they be able to rent if they charge a monthly rent of \$680? How much rent will they collect in this situation? [2]
- (b) How many apartments will they be able to rent if they charge a monthly rent of $$(650 + 10x)$? How much rent will they collect in this situation? [3]
- (c) At what monthly rate should each apartment be rented in order to maximize the total rent collected? [4]