University of Connecticut
Department of Mathematics

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Math 1071Q
Exam 1, Fall 2013

Duration: 50 minutes

Name: _____ Section: _____

Page	Points	Score
2	7	
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Total:	50	

1. Write clearly. Points may be deducted if your work is messy or your answer unclear;
2. Answer the questions in the space provided. You may use the back of the page if necessary;
3. You must show your work or explain your solution, otherwise points may be deducted;
4. No credit will be given for incorrect steps nor will credit be given for correct solutions arrived at by incorrect means;
5. If you make an unnecessary approximation in your solution to a problem, your answer will be judged on its accuracy. Points may be deducted for poor or inappropriate approximation;

1. The revenue and cost functions for a particular product are given bellow. The cost and revenue are given in dollars, and x represent the number of units.

$$\text{Revenue: } R(x) = -0.2x^2 + 158x$$
$$\text{Cost: } C(x) = 62x + 11,200$$

(a) (3 points) How many items must be sold to maximize the revenue?

Solution: Revenue, $R(x)$, is a quadratic function.
Therefore the vertex point is (h, k), where

$$h = -\frac{b}{2a}$$
$$k = c - \frac{b^2}{4a}$$

and, since the coefficient of x^2 is negative, $R(x)$ will attain a maximum of k when $x = h$.
Now, $a = -0.2, b = 158$ and $c = 0$ so

$$h = -\frac{158}{2(-0.2)} = 395.$$

Thus, 395 items must be sold to maximise the profit.

(b) (1 point) What is the profit function?

Solution: The profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -0.2x^2 + 158x - (62x + 11,200) \\ &= -0.2x^2 + 96x - 11,200. \end{aligned}$$

(c) (3 points) At what production level(s) will the company break even?

Solution: The company will break even when $R(x) = C(x)$.
So,

$$-0.2x^2 + 158x = 62x + 11,200$$
$$-0.2x^2 + 96x - 11,200 = 0$$

Using the quadratic formula we can get $x = 200$ or 280.

2. (5 points) How long will it take an investment to triple if it is continuously compounded at 10% per year?

   Recall that the future value F after t years of an principal investment P earning interest at a rate r per year compounded continuously is given by $F = Pe^{rt}$.

   Note: Your answer should either be exact or else correct to 4 decimal places.

Solution:

Here we want $F = 3P$. Since $r = 0.1$, we get

$$3P = P^{0.1t}$$

so

$$3 = e^{0.1t}.$$

Thus

$$\ln 3 = \ln(e^{0.1t}) = (0.1)\ln(e)$$

and so

$$t = \frac{ln(3)}{0.1}$$

3. The cost (in dollars) of producing x units of a certain commodity is given by

$$C(x) = 9000 + 14x + 0.1x^2.$$

   (a) (4 points) Find the average rate of change of C with respect to x when the production is changed from $x = 100$ to $x = 104$.

Solution: The average rate of change is given by

$$\frac{C(104) - C(100)}{104 - 100} = \frac{9000 + 14(104) + (0.1)(104)^2 - [9000 + 14(100) + (0.1)(100)^2]}{4}$$
$$= \frac{137.6}{4} = 34.4.$$

That is, the average cost is \$34.40 per unit.

   (b) (4 points) Find the instantaneous rate of change of C with respect to x when $x = 100$ (i.e. the *marginal cost*).

Solution: Since

$$C'(x) = 14 + 0.2x$$

we have that

$$C'(100) = 14 + 0.2(100) = 34.$$

So, the cost of producing the 101'st unit will be \$34.

4. *Note: Your answers for this question should either be exact or else correct to 4 decimal places.* Solve the following equations for x:

(a) (5 points) $3 \cdot 2^{5x+1} - 10 = 0$

Solution:

$$3 \cdot 2^{5x+1} - 10 = 0$$

so

$$2^{5x+1} = \frac{10}{3}$$

so

$$\ln(2^{5x+1}) = \ln(\frac{10}{3})$$

so

$$(5x+1)\ln 2 = \ln(\frac{10}{3})$$

so

$$x = \frac{\frac{\ln(\frac{10}{3})}{\ln 2} - 1}{5}$$

(b) (5 points) $\ln(7x-1) - \ln(x) = 5$

Solution:

$$\ln(7x-1) - \ln(x) = 5$$

so

$$\ln\left(\frac{7x-1}{x}\right) = 5$$

so

$$\frac{7x-1}{x} = e^5$$

so

$$x = \frac{-1}{e^5 - 7}$$

Note: The value $\frac{-1}{e^5-7}$ is negative. Thus, the correct conclusion to draw here is that there is *no* value for x that satisfies the given equation. However, since I did not notice this when writing the exam, you will have been given full credit if your answer is the value of x above.

5. (5 points) Evaluate

$$\lim_{x\to 5} \frac{x^2+x-30}{x^2-7x+10}.$$

Solution: Note that

$$\frac{x^2+x-30}{x^2-7x+10} = \frac{(x-5)(x+6)}{(x-5)(x-2)} = \frac{x+6}{x-2}$$

when $x \neq 5$.

Thus

$$\lim_{x\to 5} \frac{x^2+x-30}{x^2-7x+10} = \lim_{x\to 5} \frac{x+6}{x-2} = \frac{11}{3}.$$

6. Let $f(x) = -3x^4 + 9x^3$.

   (a) (3 points) Find the derivative of f at $x = -1$.

Solution: $f'(x) = -12x^3 + 27x^2$.
So, $f'(-1) = -12(-1) + 27 = 39$.

   (b) (2 points) Find the equation of the line tangent to the graph of f at $x = -1$.

Solution: $f(-1) = -3 - 9 = -12$.
Thus, the tangent line at $x = -1$ has equation

$$y - (-12) = 39(x - (-1))$$

or

$$y = 39x + 27.$$

7. (10 points) Use the **limit definition** of the derivative to find $f'(x)$ where

$$f(x) = \frac{5}{x-2}.$$

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \left(\frac{\frac{5}{x+h-2} - \frac{5}{x-2}}{h} \right) \\
&= \lim_{h\to 0} \left(\frac{\frac{5(x-2)-5(x+h-2)}{(x+h-2)(x-2)}}{h} \right) \\
&= \lim_{h\to 0} \frac{5(x-2)-5(x+h-2)}{h(x+h-2)(x-2)} \\
&= \lim_{h\to 0} \frac{-5}{(x+h-2)(x-2)} \\
&= \frac{-5}{(x-2)^2}.
\end{aligned}$$