University of Connecticut
Department of Mathematics

MATH 1070 SAMPLE EXAM 1

SOLUTIONS

Exam 1 will cover sections 1.1, 1.2, 3.1, 3.2, 3.3, 4.1, 4.2, 4.3, 4.4, 4.5, 5.1 and 5.2. This sample exam is intended to be used as one of several resources to help you prepare. The coverage of topics is not exhaustive, and you should look through all examples from lectures (or videos), quizzes, and homework as these will all be relevant. The wealth of problems in our text is also a good resource for practice with this material.

- The exam is a closed notes, closed book exam. You can not receive aid on this exam from anyone. Approved calculators are allowed, but there is no sharing of calculators!
- Some partial credit may be given depending on the correctness of the work submitted. You must show all work and calculations needed to reach your answers. Just using a calculator is not sufficient for credit.
- Please make sure to attend the exam that you signed up for at the beginning of the term. The room for your exam can be found on the common course webpage.

1. A diner finds that there is demand for 96 burgers when they are priced at \$5.15 each, while the demand increases to 120 when they are priced at \$3.95 each. Assume linear supply and demand models.

   (a) Find the demand equation $p = D(x)$.

slope = $m = \frac{5.15 - 3.95}{96 - 120} = -0.05$ (2)

$p = mx + b$ OR $p - p_1 = m(x - x_1)$

$5.15 = -0.05(96) + b$ — (2) — $p - 5.15 = -0.05(x - 96)$

$9.95 = b$ $p = -0.05x + 4.8 + 5.15$

$p = -0.05x + 9.95$ — (1) — $p = -0.05x + 9.95$

   (b) If the supply equation is $p = S(x) = 0.1x + 2.45$, find the equilibrium price and quantity.

$0.1x + 2.45 = -0.05x + 9.95$ (1)

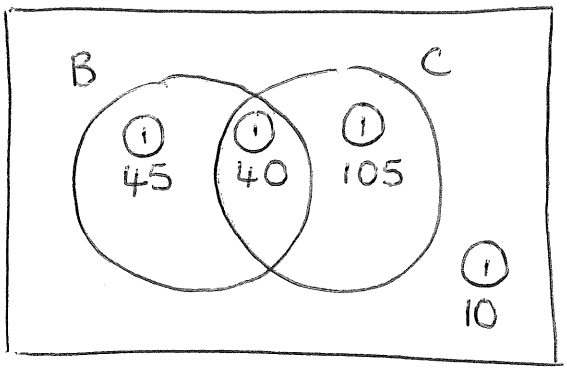
$0.15x = 7.5$ (1)

$x = 50$ (1)

$p = 0.1(50) + 2.45 = 7.45$ (1)

2. In a survey of 200 people, it was found that 190 own a car or a bike, while 145 own a car, and 85 own a bike.

   (a) Represent this information in a Venn diagram.



   (b) If someone is selected randomly from this group of people, what is the probability that she or he owns a bike but not a car? [3]

$$\frac{45}{200}$$

3. We toss a fair six-sided die twice, and record the number that is showing on each toss.

   (a) How many outcomes are there in the sample space?

$6 \times 6 = 36$ (1) (1)

   (b) What is the probability that the die shows 5 at least once?

(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(1,5) (2,5) (3,5) (4,5) (6,5) } (2)

$\frac{11}{36}$ (1)

   (c) What is the probability of the die does not show 5?

$36 - 11 = 25$ (2)

$\frac{25}{36}$ (1)

4. A family is taking a photo of their 3 dogs and 2 cats arranged in a row.

   (a) How many ways are there to arrange the 5 pets in a row?

$5!$ (2)

   (b) How many ways are there to arrange the 5 pets in a row with if the 2 cats must be next to each other?

(C1 C2) (D1) (D2) (D3)

4 groups

(2) $4! \, 2!$ (1) (1) for multiplying.

$4!$: arrange 4 gps

$2!$: arrange 2 cats within their gp.

5. Suppose a firm manufacturing microwaves has a fixed cost of \$4,000 and an additional cost of \$75 for each microwave produced. Assume linear cost and revenue models.

   (a) Find the cost equation.

$$C = 4000 + 75x \quad (1) \quad (1)$$

   (b) If the profit from selling 100 microwaves is \$7,250 find the revenue equation.

$x = 100$: $C = 4000 + 75(100) = 11500$

$P = R - C$ (3)

$7250 = R - 11500$

$18750 = R$

$R = px$

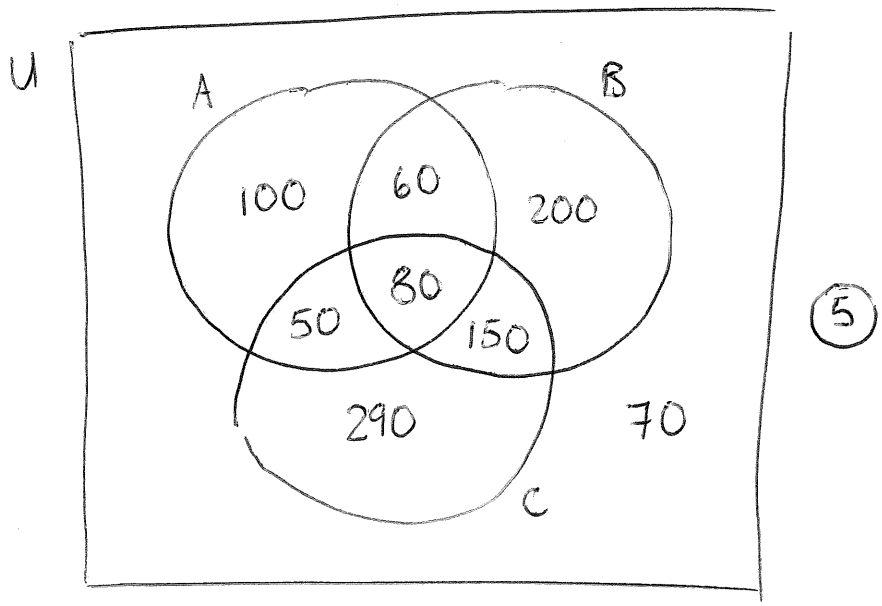
$18750 = p(100)$ (2)

$187.5 = p$

$R = 187.5x$ (1)

6. We are given the following information: $n(U) = 1000$, $n(A) = 290$, $n(B) = 490$, $n(C) = 570$, $n(A \cap B) = 130$, $n(A \cap C) = 140$, $n(B \cap C) = 230$, and $n(A \cap B \cap C) = 80$.

(a) Represent this information in a Venn diagram.



(5)

(b) Find $n(A^c \cap C)$.

$290 + 150 = 440$

(1) (1)

(c) Find $n(A \cap B^c \cap C)$.

50 (2)

(d) Find $n((A \cap C) \cup B)$.

$50 + 60 + 80 + 150 + 200 = 540$

(2) (1)

7. A farmer has 12 acres of land on which to plant turnips and potatoes. Each acre of turnips will yield \$400 in profit, while each acre of potatoes will yield \$300 in profit. If the farmer wants to have at least twice as many acres of potatoes as acres of turnips, how many acres of each crop should be planted to maximize revenue?

   (a) Express this linear programming problem by defining appropriate variables, writing the objective function, and writing the corresponding constraints.

x = acres of turnips, y = acres of potatoes (1)

objective: $R = 400x + 300y$ (1)

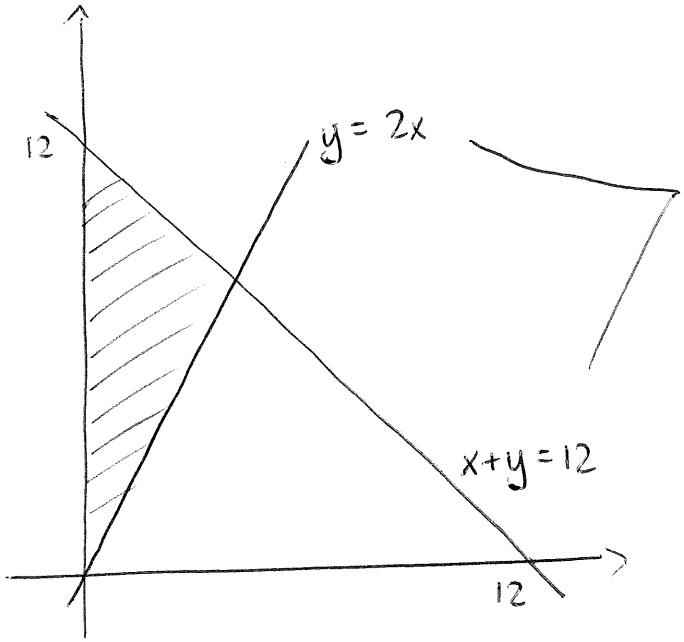
land: $x + y \leq 12$ (1)

ratio: $y \geq 2x$ (1)

non-negativity: $x, y \geq 0$ (1)

   (b) Using the equations from part (a) graph the feasible region and find the corner points.

(3) graph



$x + 2x = 12$

$3x = 12$

$x = 4$ $y = 2(4) = 8$ } (3)

(4, 8)

(0, 12) (1)

(0, 0) (1)

(c) Determine how many acres of each crop should be planted to maximize the revenue.

	x	y	$R = 400x + 300y$
(1)	0	0	0
(1)	0	12	3600
(1)	4	8	4000

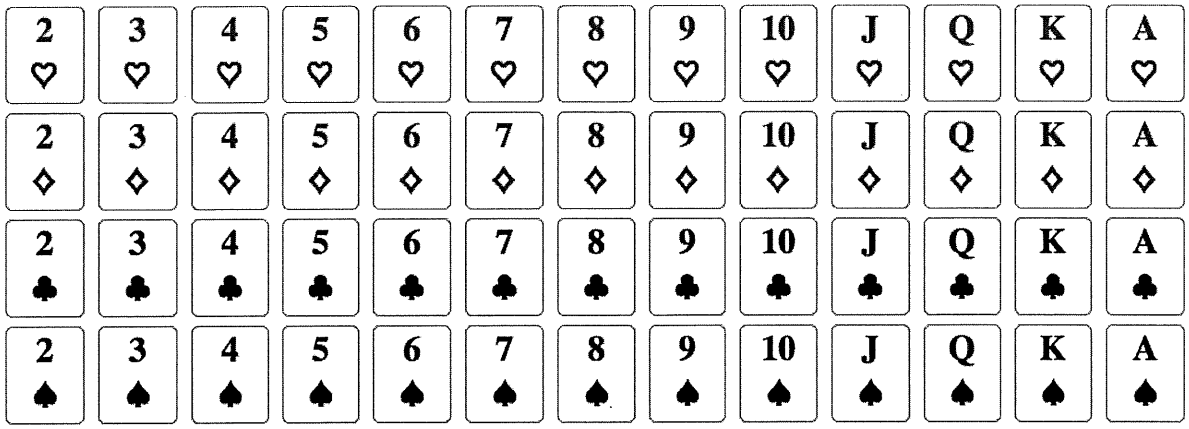
← plant 4 acres turnips, 8 acres potatoes (1)

8. In a contest with 20 participants, there will be one 1st prize, two identical 2nd prizes, four identical 3rd prizes, and three identical wild card prizes. The 1st, 2nd and 3rd prizes must all go to different people. The wild card prizes can go to anyone, even those that have won a 1st, 2nd or 3rd prize. How many different ways are there to distribute the prizes?

(1) (2) (2) (2)
$C(20,1) \cdot C(19,2) \cdot C(17,4) \cdot C(20,3)$

(1) for multiplying

9. Suppose we draw a 5-card hand from a standard 52-card deck, depicted below.



(a) How many different hands contain a pair of 7s, a different pair, and one card of a different value, e.g. two 7s, two Kings, and one 10?

(1) (1) (1) (1) (1) (1) for multiplying

$C(4,2) \cdot C(12,1) \cdot C(4,2) \cdot C(11,1) \cdot C(4,1)$

select two 7s — select value of other pair — select cards of that value — select value of last card — select last card.

(b) How many different hands contain three cards of one suit, and a pair of cards of a different suit, e.g. three Diamonds and two Spades?

(1) (1) (1) (1) (1) for multiplying

$C(4,1) \cdot C(13,3) \cdot C(3,1) \cdot C(13,2)$

select suit for 3 of a kind — select 3 cards of that suit — select other suit — select 2 cards of that suit.

10. Let the sample space be $S = \{a, b, c, d\}$. How many events are there?

ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$

$\{a,b\}$, $\{a,c\}$, $\{a,c\}$, $\{a,d\}$, $\{b,c\}$, $\{b,d\}$, $\{c,d\}$ } (3)

$\{a,b,c\}$, $\{a,b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$

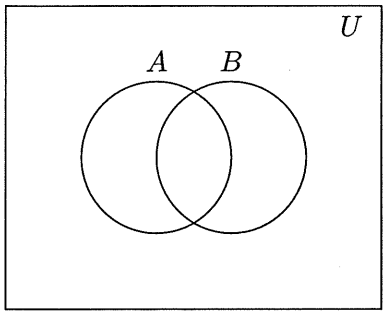
$\{a,b,c,d\}$

16 (1)

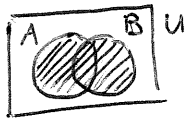
OR $2^4 = 16$

(3) (1)

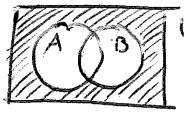
11. Using the following diagram shade the sets indicated. (You may wish to recopy this diagram for each set).

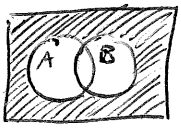


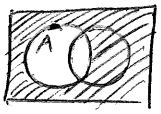
A. $A \cup B$
B. $A^c \cup B^c$
C. $A^c \cap B^c$
D. $(A \cup B)^c$
E. $U \cap A^c$
F. $U \cup B$
G. $A^c \cup B^c \cup (A \cap B)$
H. $A \cap A^c$

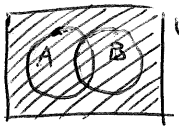
A.  (1)

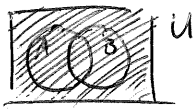
B.  (2)

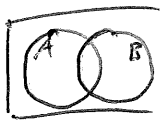
C.  (2)

D.  (1)

E.  (2)

F.  (2)

G.  (2)

H.  (empty) (2)